ENGINEERING FORMULAS SERIES

CIVIL ENGINEERING

Theory of Structures
Reinforced Concrete Design
Structural Steel Design
Timber Design
Surveying and Transportation
Fluid Mechanics & Hydraulics
Geotechnical Engineering

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THEORY OF STRUCTURES

DEFLECTION OF BEAMS

The deflection of a beam is usually expressed in terms of its deflection from its original unloaded position. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the elastic curve of the beam.

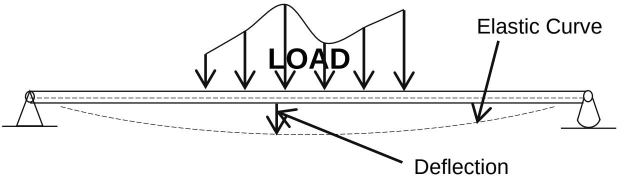


Figure: Elastic Curve

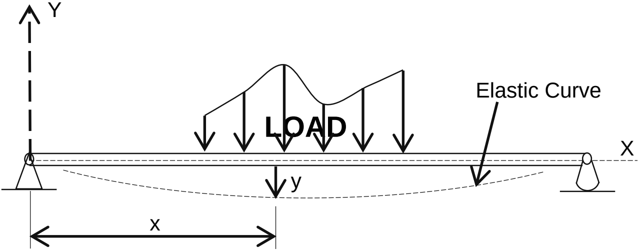
Methods of Determining Beam Deflections:

Numerous methods are available for the determination of beam deflections. These methods includes

1. Double – integration method
2. Area – moment method
3. Strain – energy method (Castigliano's Theorem)
4. Three – moment equation
5. Conjugate – beam method
6. Method of superposition
7. Virtual work method

DOUBLE INTEGRATION METHOD

The double integration method is a powerful tool in solving deflection and of a beam at any point because we will be able to get the equation of the elastic curve.



In Calculus, the radius of curvature of a curve y = f(x) is given as $\rho = \dfrac{[1+(dy/dx)^2]^{3/2}}{\left|d^2y/dx^2\right|}$. Deflection of beams are so small such that the slope of the elastic curve is very small (dy / dx = 0), and squaring this expression the value becomes practically negligible, hence $\rho = \dfrac{1}{d^2y/dx^2} = \dfrac{1}{y''}$.

In Strength of Materials, the radius of curvature of a beam subject to bending is $\rho = \dfrac{EI}{M}$.

Therefore, $\dfrac{EI}{M} = \dfrac{1}{y''}$.

$$\boxed{y'' = \frac{M}{EI} = \frac{1}{EI}\,M}$$

If EI is constant, the equation may be written as:

$$\boxed{E\,I\,y'' = M}$$

Where x and y are the coordinates shown in the figure above, y is the deflection of the beam at any distance x. E is the modulus of elasticity of the beam, I represent the moment of inertia about the neutral axis, and M represents the bending moment at a distance x from the end of the beam. The product EI is called the flexural rigidity of the beam.

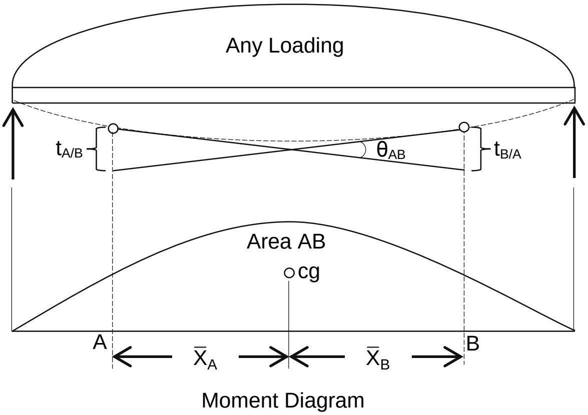
The first integration y' yields the slope of the elastic curve and the second integration y gives the deflection of the beam at any distance x.

The resulting solution must contain two constants of integration since EIy'' = M is of second order. These two constants must be evaluated from known conditions concerning the slope deflection at certain points of the beam. For instance, in the case of a simply supported beam with rigid supports, at x = 0 and x = L, the deflection y = 0, and in locating the point of maximum deflection we simply set the slope of the elastic curve y' to zero.

AREA – MOMENT METHOD

Another method of determining the slopes and deflections in beams is the area – moment method which involves the area of the moment diagram.

Conduct the two points A and B shown in the figure:



<u>Theorems on Area – Moment Method</u>

<u>Theorem 1</u>

The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1 / EI multiplied by the area of the moment diagram between these two points.

$$\boxed{\theta_{AB} = \frac{1}{EI}(\text{Area}_{AB})}$$

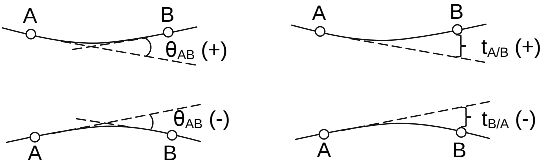
Theorem 2

The deviation of any point B relative to the tangent drawn to the elastic curve at any point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

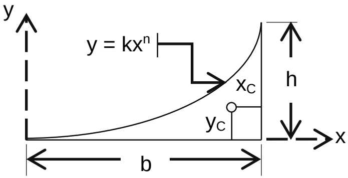
$$t_{B/A} = \frac{1}{EI}\,(Area_{AB})\,\bar{X}_B$$

$$t_{A/B} = \frac{1}{EI}\,(Area_{AB})\,\bar{X}_A$$

Rules of Sign:



Area and Centroid of Common Moment Diagram Shape (Spandrel)



$$A = \frac{1}{n+1}\, bh$$

$$X_G = \frac{1}{n+2}\, b \;; \qquad Y_G = \frac{n+1}{4n+2}\, h$$

STRAIN ENERGY METHODS

These are various techniques (aside from the previous discussion in this section) for finding deformations and values of indeterminate reaction. These techniques are based upon geometric considerations.

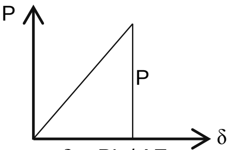
Strain energy method is based upon relations between the work done by external forces and the internal strain energy stored within the body during the deformation process. This process is more general and more powerful than the various geometric approaches.

Strain Energy, U

When an external force acts upon an elastic body and deforms it, the work done by the forces is stored within a body in the form of strain energy. The strain energy is always a scalar quantity.

For a straight bar subjected to a normal force P (tension or compression), the internal strain energy U is

$$U = \tfrac{1}{2}\, P\, \delta$$



$\delta = PL / AE$

$$U = \frac{1}{2}\,P\,(PL\,/\,AE)$$

$$\boxed{U = \frac{P^2 L}{2\,A\,E}}$$

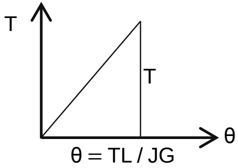
If the axial force P varies along the length of the bar

$$\boxed{U = \int_0^L \frac{P^2 dx}{2\,A\,E}}$$

For a circular shaft of length L subjected to a torque T, the internal strain energy U is

$$U = \frac{1}{2}\,T\,\theta$$

$$U = \frac{1}{2}\,T\,(TL\,/\,JG)$$



$$\boxed{U = \frac{T^2 L}{2\,J\,G}}$$

If the torque T acts along the length of the bar, the total strain energy is

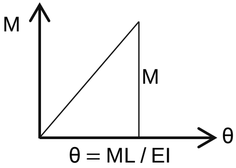
$$\boxed{U = \int_0^L \frac{T^2 dx}{2\,J\,G}}$$

For a bar length L subject to a bending moment M, the internal strain energy U is

$$\frac{1}{\rho} = \frac{M}{E\,I} = \frac{\theta}{L}$$

$$\theta = \frac{ML}{EI}$$

$$U = \frac{1}{2} M\theta = \frac{1}{2} M(ML / EI)$$



$$\boxed{U = \frac{M^2 L}{2\,E\,I}}$$

If the bending moment varies along the length of the bar, the total internal strain energy is

$$\boxed{U = \int_0^L \frac{M^2 dx}{2\,E\,I}}$$

Where θ is central angle subtended by the circular arc of radius ρ.

CASTIGLIANO'S THEOREM

The displacement of an elastic body under the point of application of any force, in the direction of that force, is given by the partial derivative of the total internal strain energy with respect to that force.

$$\delta_n = \frac{\partial U}{\partial P_n}$$

For a body subject to combined axial, torsional, and bending effects, Castigliano's theorem is conveniently expressed as

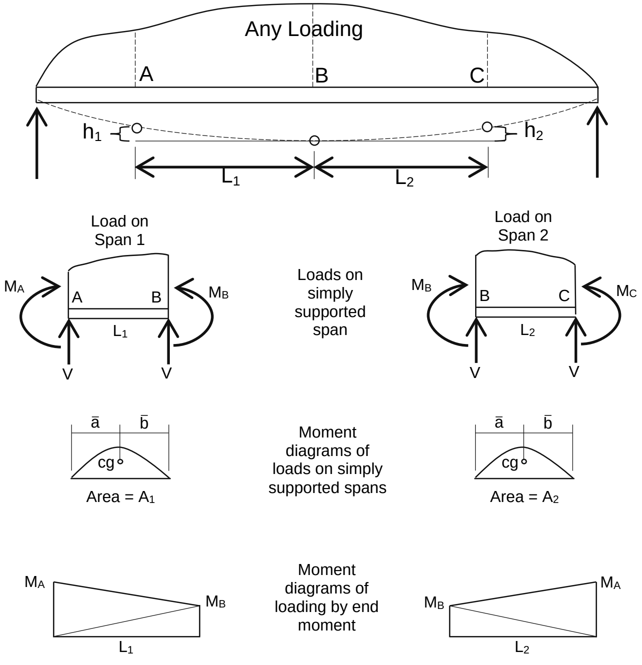
$$\delta_n = \int \frac{P\,(\partial P / \partial P_n)dx}{AE} + \int \frac{T\,(\partial T / \partial T_n)dx}{JG} + \int \frac{M\,(\partial M / \partial P_n)fx}{EI}$$

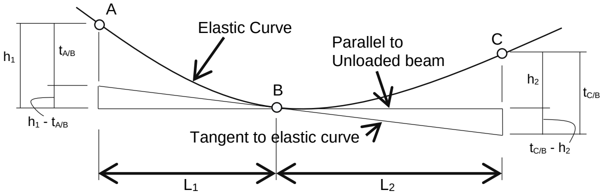
For a body composed of a finite number of elastic subbodies, these integrals may be replaced by finite summations.

If rotation is required, P_n may be replaced by m_n, which is the applied couple at a point in question.

THREE MOMENT EQUATION

The three moment equation gives us the relation between the moments between any three points in a beam and their relative vertical distances or deviations. This method is widely used in finding the reactions in a continuous beam.





From proportions between similar triangles:

$$\frac{h_1 - t_{A/B}}{L_1} = \frac{t_{C/B} - h_2}{L_2}$$

$$\frac{t_{A/B}}{L_1} + \frac{t_{C/B}}{L_2} = \frac{h_1}{L_1} + \frac{h_2}{L_2} \qquad \rightarrow (1)$$

$$t_{A/B} = \frac{1}{E_1 I_1}\,(\text{Area})_{AB}\,\overline{X_A}$$

$$t_{A/B} = \frac{1}{E_1 I_1}\left[A_1\bar{a}_1 + \frac{1}{2}M_A L_1 \text{ x } \frac{1}{3}L_1 + \frac{1}{2}M_B L_1 \text{ x } \frac{2}{3}L_1\right]$$

$$t_{A/B} = \frac{1}{6E_1 I_1}\left[6A_1\bar{a}_1 + M_A L_1{}^2 + 2M_B L_1{}^2\right]$$

$$t_{A/B} = \frac{1}{E_2 I_2}\,(\text{Area})_{BC}\,\overline{X}_C$$

$$t_{A/B} = \frac{1}{E_1 I_1}\left[A_2\bar{b}_2 + \frac{1}{2}M_B L_2 \text{ x } \frac{2}{3}L_2 + \frac{1}{2}M_C L_2 \text{ x } \frac{1}{3}L_2\right]$$

$$t_{A/B} = \frac{1}{6E_2 I_2}\left[6A_2\bar{b}_2 + 2M_B L_2{}^2 + M_C L_2{}^2\right]$$

Substitute $t_{A/B}$ & $t_{C/B}$ to Eq. (1):

$$\frac{1}{6E_1I_1}\left[\frac{6A_1\bar{a}_1}{L_1} + M_AL_1 + 2M_BL_1\right]$$

$$+\frac{1}{6E_2I_2}\left[\frac{6A_2\bar{b}_2}{L_2} + 2M_BL_2 + M_CL_2\right] = \frac{h_1}{L_1} + \frac{h_2}{L_2}$$

Simplify

$$\boxed{\begin{gathered}\frac{M_AL_1}{E_1I_1} + 2M_B\left(\frac{L_1}{E_1I_1} + \frac{L_2}{E_2I_2}\right) + \frac{M_CL_2}{E_2I_2} \\ + \frac{6A_1\bar{a}_1}{E_1I_1L_1} + \frac{6A_2\bar{b}_2}{E_2I_2L_2} = 6\left(\frac{h_1}{L_1} + \frac{h_2}{L_2}\right)\end{gathered}}$$

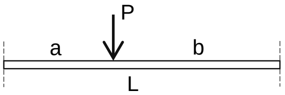
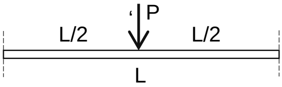
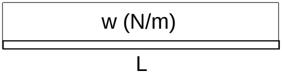
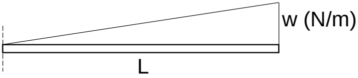
If E is constant this equation becomes,

$$\boxed{\begin{gathered}\frac{M_AL_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_CL_2}{I_2} \\ + \frac{6A_1\bar{a}_1}{I_1L_1} + \frac{6A_2\bar{b}_2}{I_2L_2} = 6E\left(\frac{h_1}{L_1} + \frac{h_2}{L_2}\right)\end{gathered}}$$

If E and I are constant then,

$$\boxed{\begin{gathered}M_AL_1 + 2M_B(L_1 + L_2) + M_CL_2 \\ + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 6EI\left(\frac{h_1}{L_1} + \frac{h_2}{L_2}\right)\end{gathered}}$$

Values of $\frac{6A\bar{a}}{L}$ and $\frac{6A\bar{b}}{L}$ of Common Loadings:

$\frac{6A\bar{a}}{L}$	$\frac{6A\bar{b}}{L}$
	
$\frac{Pa}{L}\left(L^2 - a^2\right)$	$\frac{Pb}{L}\left(L^2 - b^2\right)$
	
$\frac{3}{8}PL^2$	$\frac{3}{8}PL^2$
	
$\frac{wL^3}{4}$	$\frac{wL^3}{4}$
	

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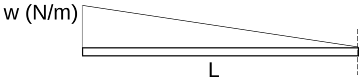
<tr><td colspan="2">
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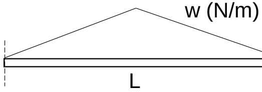
<tr><td colspan="2">
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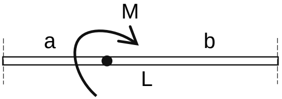
<tr><td>$5\frac{wL^3}{32}$</td><td>$5\frac{wL^3}{32}$</td></tr>
<tr><td colspan="2">
</td></tr>
<tr><td>$-\frac{M}{L}\ (3a^2 - L^2)$</td><td>$+\frac{M}{L}\ (3b^2 - L^2)$</td></tr>
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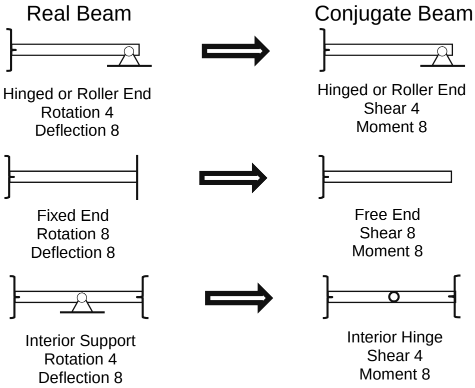
CONJUGATE BEAM METHOD

Conjugate beam method determines the slope and deflections of a real beam by calculating the shears and moments of a fictitious beam called the conjugate beam loaded with the M/EI diagram.

Slope on real beam = Shear on conjugate beam
Deflection on real beam = Moment on conjugate beam

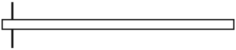
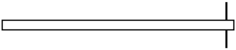
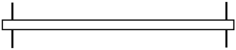
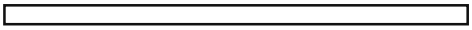
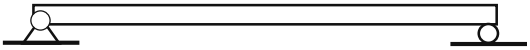
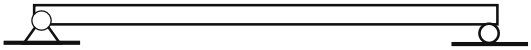
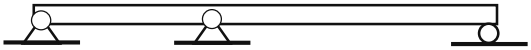
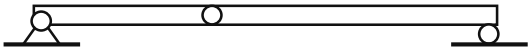
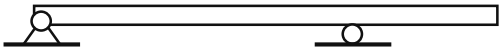
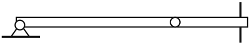
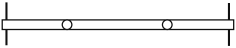
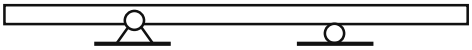
Properties of Conjugate Beam

1. The length of a conjugate beam is always equal to the length of the actual beam.
2. The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
3. A simple support for the real beam remains simple support for the conjugate beam.
4. A fixed end for the real beam becomes free end for conjugate beam.
5. The point of zero shear for the conjugate beam corresponds to a point of zero slope for the real beam.
6. The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.



Interior Hinge
Shear 4
Moment 8

Interior Support
Rotation 4
Deflection 8

Real Beam	Conjugate Beam
	
	
	
	
	
	

Equilibrium of Conjugate Beams

Conjugate beams are always statically determinate; hence the reactions, moments and shears of the conjugate beam are easily computed by statics. In some instances, the conjugate may appear to be unstable due to missing reactions (as for a fixed ended beam), but it can be observed that the positive and negative areas of the M/EI diagrams due to the actual loads balances the conjugate beam.

Example

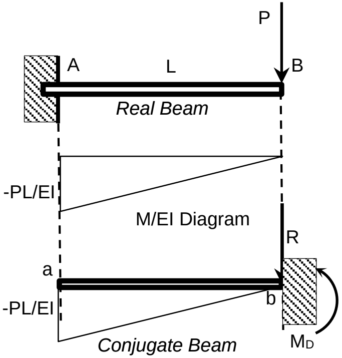
Deflection of cantilever beam With concentrated load at the Free end.

$\theta_B = V_b = R = 1/2\ (L)(-PL/EI)$

$$\theta_B = -\frac{PL^2}{2EI}$$

$\delta_B = M_b = 1/2\ (L)(-PL/EI)(2L/3)$

$$\delta_B = -\frac{PL^3}{3EI}$$



VIRTUAL WORK METHOD

Deflection and rotation at any point on a beam, truss, or frame can be obtained using Virtual work method.

Virtual Work Equation for Beams and Frames

The deflection in any direction at a point on a beam or frame can be obtained by applying a unit load at that point and applying the formula

$$\boxed{\delta_n = \int_0^L \frac{M m_n dx}{E\,I}}$$

Where M is the bending moment at the element under consideration due to applied loadings, and m_n is the bending moment due to unit load applied at the point where the deflection is required.

If the rotation at a point is required, apply a unit couple at a point and use the equation

$$\boxed{\theta = \int_0^L \frac{M m_n dx}{E\,I}}$$

Where M is the bending moment at the element under consideration due to applied loadings, and m_n is the bending moment due by the unit couple applied at the point where the rotation is required.

Virtual Work Equation Due to Temperature Change

The virtual work equation due to temperature change is:

$$\boxed{\delta = u\,\alpha\,(\Delta T)L}$$

Where u is the stress in the member due to unit load, α is the coefficient of thermal expansion of the member, and ΔT is the temperature change.

Virtual Work for Trusses

The virtual work for trusses is:

$$\boxed{\delta = \sum \frac{SUL}{AE}}$$

Where S the stress in a member due to actual loads, L is the length of the member, A is the cross – sectional of the member, E is the modulus of elasticity, and U is the stress in the member due to the virtual unit load.

INDETERMINATE BEAMS

As discussed in the previous section, indeterminate beams are those beams in which the number of reactions exceeds the number of equation in static equilibrium. The degree of indeterminacy is the difference between the number of reactions (forces and moments) to the number of equations in static equilibrium.

$$\boxed{\text{Degree} = \text{Number of reactions} - \text{Number of equilibrium equations}}$$

In such a case, it is necessary to supplement the equilibrium equations with additional equations arising from the deformation of the beam.

Stability and Determinacy of Structures

In general, structures may be stable or unstable. If a structure is stable, it may be determinate or indeterminate.
For a coplanar structure there are at least three equations of equilibrium that can be made ($\sum F_V = 0$, $\sum F_H = 0$, & $\sum M = 0$). An additional equation is made for every internal hinge present due to fact that the moment at this point is zero.
Reactions and Equations

- A hinge support has two (2) support,
- A roller support has one reaction,
- A fixed (fully restrained) support has three (3) reactions,
- There are at least three (3) equations in every structure,
- An internal hinge provides one additional equation.

Stability of Structures

A structure is geometrically unstable if there are fewer reactive forces than equations of equilibrium; or if there are enough reactions, instability occurs if the lines of action of these forces intersect at a common point.

Determinacy of Structure

A structure is statically determinate if the number of equations equals the number of external reactions. If the number of external reactions exceeds the number of equations, the structure becomes statically indeterminate.

The degree of determinacy is the difference between the number of reactions and the number of equations that can be made.

Degree of indeterminacy = Number of reactions – Number of equations

Types of Indeterminate Beams

There are several types of indeterminate structure exist in practice. The following diagrams will illustrate the nature of indeterminate beams.

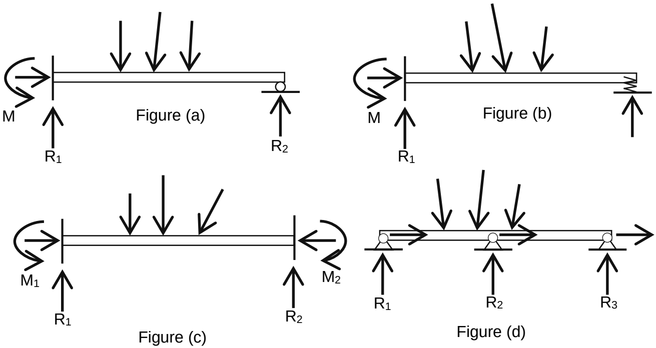


Figure (a) is called propped beam or supported cantilever having three unknowns R_1, R_2, H and M. This is indeterminate to the first degree.

Figure (b) is fixed at one end and has a flexible spring like support at the other end. In the case of a simple linear spring, the flexible support exerts a force proportional to the beam deflection at that point.

Figure (c) is fixed or clamped at both ends and is a perfectly restrained beam. This beam is indeterminate to the third degree.

Figure (d) has six unknown reaction. This type of beam that rests on more than two supports is called a continuous beam. This beam can be solved using the three – moment equation or moment distribution method.

THE THREE – MOMENT EQUATION

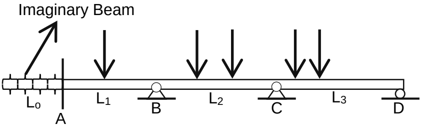
The three – moment equation is an effective equation in solving continuous beams. In that equation, the three supports of a continuous beam may be selected as the three points A, B, and C. If these supports are rigid, the values of h_1 and h_2 is zero. With E and I constant, the equation may be written in the form.

$$\boxed{M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 0}$$

The values of $6A\bar{a}/L$ and $6A\bar{b}/L$ are as given in table.

For the beam shown in Figure (d), the moments M_A and M_C are zero thus there is only one unknown, M_B. Using three – moment equation with A, B, and C as the three points, the moment M_B can found easily.

For the beam shown below, there are three unknown moments (M_A, M_B, & M_C) since M_D is zero. Three equations will therefore be needed to solve the beam. One equation can be obtained by taking points A-B-C, a second equation is by taking points B-C-D. The third equation can be obtained by extending an imaginary beam beyond the restrained end A, and taking points O-A-B, with all terms that refer to the imaginary span have zero values. Thus for beams with restrained ends, extend an imaginary beam to complete the necessary equations.



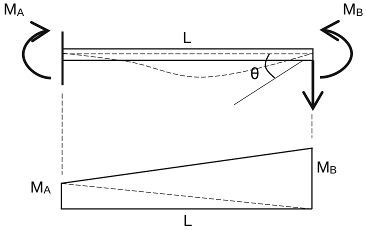
Note: the need to use this imaginary span will only arise if there are fixed end.

MOMENT DISTRIBUTION METHOD

Moment distribution is based on a method of successive approximations popularized by Hardy Cross. This method is applicable to all types of rigid frame analysis.

Carry – Over Moment

Carry – over moment is defined as the moment induced at the fixed end of a beam by the action of a moment applied at the other end. Consider the beam shown. When a moment is applied at B and flexes the beam it induces a wall moment M_A.



Since the deviation of B from the tangent through A is zero then,

$$EI\ t_{B/A}=(Area)_{AB}\ \bar{x}_B$$

$$0=\left(\frac{1}{2}\ M_A\ L\right)(2L/3)+\left(\frac{1}{2}\ M_B\ L\right)(L/3)$$

$$M_A=-\frac{1}{2}\ M_B$$

Therefore, the moment applied at B carries over to the fixed end A, a moment that is half the amount and opposite sign.

Beam Stiffness

Beam stiffness is the moment required by the simply supported end of a beam to produce a unit rotation of that end, the other end being rigidly fixed.

From the beam shown in page 25, the rotation of B relative to the tangent through A is

$$E\,I\,\theta = (\text{Area})_{AB}$$

$$E\,I\,\theta = \tfrac{1}{2}\,M_A\,L + \tfrac{1}{2}\,M_B\,L$$

but $$M_A = -\frac{1}{2}\,M_B$$

$$E\,I\,\theta = \frac{1}{2}\left(-\frac{1}{2}\,M_B\right)L + \frac{1}{2}\,M_B\,L$$

$$M_B = 4E\,I\,\theta\,/L$$

When θ equals 1 radian, M_B is called as the beam stiffness and it varies with the ratio I/L and E. Beam stiffness is denoted as K and hence,

$$\boxed{\text{Absolute } K = \frac{4\,E\,I}{L}}$$

In many structures, the value of E remains constant and only a relative measure of resistance is required. The relative beam stiffness is

$$\boxed{\text{Relative } K = \frac{I}{L}}$$

If I is not specified, it is convenient to take I as the common multiple of the span lengths.

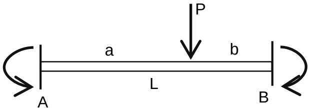
Fixed – End Moments (FEM)

In the moment distribution method, we first assume the individual spans to be fully restrained at both ends, then we compute the fixed end moments. As a rule of sign **counterclockwise moments acting on the beam (clockwise reaction) are considered positive, and clockwise moments acting on the beam (counterclockwise reaction) are considered negative**. For beams with vertical downward loads only, negative moment occurs at the left end and positive moment at the right end.

The following are the fixed end moments for common types of loading to be used with moment distribution.

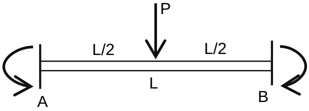
$$FEM_{AB} = \frac{Pab^2}{L}$$

$$FEM_{BA} = + \frac{Pba^2}{L^2}$$



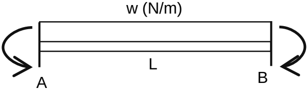
$$FEM_{AB} = - \frac{PL}{8}$$

$$FEM_{BA} = + \frac{PL}{8}$$



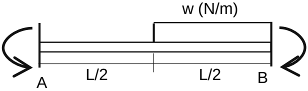
$$FEM_{AB} = -\frac{wL^2}{12}$$

$$FEM_{BA} = +\frac{wL^2}{12}$$



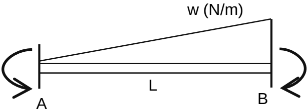
$$FEM_{AB} = -\frac{5wL^2}{192}$$

$$FEM_{BA} = -\frac{11wL^2}{192}$$



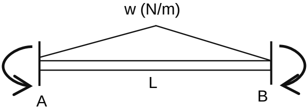
$$FEM_{AB} = -\frac{wL^2}{30}$$

$$FEM_{BA} = +\frac{wL^2}{20}$$

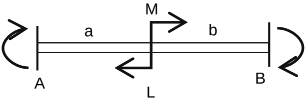


$$FEM_{AB} = -\frac{5wL^2}{96}$$

$$FEM_{BA} = +\frac{5wL^2}{96}$$



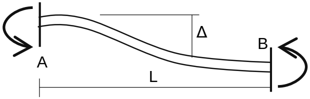
$$FEM_{AB} = +\frac{Mb}{L}\left(\frac{3a}{L} - 1\right)$$



$$FEM_{BA} = + \frac{Ma}{L}\left(\frac{3b}{L} - 1\right)$$

$$FEM_{AB} = - \frac{6EI\Delta}{L^2}$$

$$FEM_{BA} = - \frac{6EI\Delta}{L^2}$$



Distribution Factor, DF

In a continuous beam, the moments between any two adjacent spans are generally not equal. The unbalanced moment must be distributed to the other end of each span. The ratio of distribution to any beam is called the distribution factor, DF and is defined by,

$$DF = \frac{K}{\sum K}$$

At fixed – end, DF = 0

At hinged or roller end, DF = 1

where K is the stiffness factor and $\sum K$ is the sum of the stiffness factors for adjacent beams. Ff the beams are of the same material, only relative K need be used.

Steps of using moment distribution method:

1. Assume that all supports are fixed or locked and compute the fixed end moments.

2. Unlock each support and distribute the unbalance moment at each one to each adjacent span using the distribution factor DF.
3. After distribution, carry – over one – half of the moment in step 2 with the same sign, to the other end of each span.
4. Repeat steps 2 and 3 until the carry – over moment becomes distributing the rest of the moments.

Hint: For faster distribution, first distribute the joints with large unbalanced moment (especially those hinge or roller end), and carry – over the moment to the interior support, then begin distributing the rest of the moments.

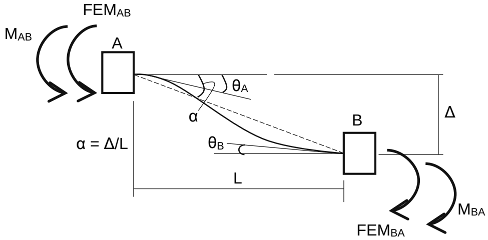
Modified K

For continuous beams with hinge or roller ends, the final moment at that end is zero. The distribution of moment will become easier if we multiply the beam stiffness K of the span containing that support by ¾, which would eliminate any further distribution of moment on that support. Do not apply this for fixed support.

SLOPE – DEFLECTION METHOD

The slope – deflection method was introduced by George A. Maney of the University of Minnesota in the year 1915. In this method, the moment at the end of each member is expressed in terms of the (a) fixed – end moment due to external loads, (b) the rotation of the tangent at the end of each elastic curve, and (c) the rotation of the chord joining the ends of the elastic curve.

Slope – Deflection Equation



With reference to the figure shown above.

$$M_{AB} = FEM_{BA} + K\ (2\theta_A + \theta_B - 3\alpha)$$

$$M_{BA} = FEM_{BA} + K\ (\theta_A + 2\theta_B - 3\alpha)$$

$$\alpha = \Delta / L$$

$$\text{Absolute } K = \frac{2EI}{L}; \qquad \text{Relative } K = I / L$$

If A and B are points of support such that Δ = 0, the equation becomes:

$$M_{AB} = FEM_{AB} + K\ (2\theta_A + \theta_B)$$

$$M_{BA} = FEM_{BA} + K\ (\theta_A + 2\theta_B)$$

The sign of the fixed end moments is the same as that used in the moment distribution method.

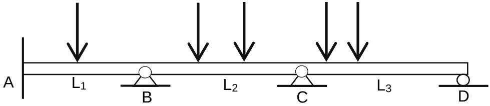
In general, the slope – deflection equation can be expressed as:

$$\boxed{M_N = FEM_N + K\ (2\theta_N + \theta_F - 3\alpha)}$$

Where:

M_N= internal moment in the near end of the span
FEM_N = fixed – end moment at the near end support
θ_N, θ_F = near – and far – end slopes of the span at support

Application of Slope – Deflection Equations to Continuous Beams.



Each member of the beam is considered individually and fixed – end moments are computed. One equation for moment is computed at each end of the member. For the continuous beam shown, for span AB, equations for M_{AB} and M_{BA} are written; for span BC, equations for M_{BC} and M_{CB} are written, and so on. All these moment equation are expressed in terms of the unknown values of θ at the supports. The unknown θ can be solved by the following support conditions.

1. The rotation θ is 0 for fixed ends, such as θ_A in the beam shown above.
2. The moment M is zero at simple ends of the beam such M_D in the beam shown above.
3. The sum of two moment at an interior support must be zero, i.e..

$$M_{BA} + M_{BC} = 0$$

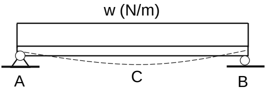
$$M_{CB} + M_{CD} = 0$$

SIMPLE AND CANTILEVER BEAM FORMULAS

$$M_{max} = M_{mid} = \frac{wL^2}{8}$$

$$\delta_{max} = \delta_{mid} = \frac{5wL^4}{384EI}$$

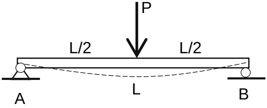
$$\theta_A = \theta_B = \frac{wL^3}{24EI}$$



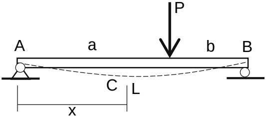
$$M_{max} = M_{mid} = \frac{PL}{4}$$

$$\delta_{max} = \delta_{mid} = \frac{PL^3}{48EI}$$

$$\theta_A = \theta_B = \frac{PL^2}{16EI}$$



$$M_{max} = M_P = \frac{Pab}{L}$$

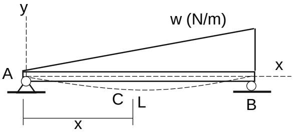


$$x = \frac{\sqrt{L^2 - b^2}}{3}$$

$$\delta_{max} = \delta_C = \frac{Pb\,(L^2 - b^2)^{3/2}}{9\sqrt{3}\ EIL}$$

$$\delta_{mid} = \frac{Pb}{48\ EI}\left(3L^2 - 4b^2\right) \quad \text{when } a>b$$

$$\delta_{mid} = \frac{2.5wL^4}{384EI}$$

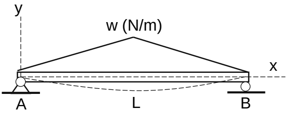


Location of max. δ
$x = 0.481\ L$

$$\theta_B = \frac{8wL^3}{360EI}\ ;\ \theta_A = \frac{7wL^3}{360EI}$$

$$EIy = \frac{wx}{360L}\left(7L^4 - 10L^2x^2 + 3x^4\right)$$

$$M_{max} = \frac{wL^2}{12}$$



$$\theta_A = \theta_B = \frac{5wL^3}{192EI}$$

$$\delta_{max} = \delta_{mid} = \frac{wL^4}{120EI}$$

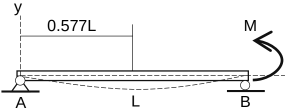
$$EIy = \frac{wx}{960L}\left(25L^4 - 40L^2x^2 + 16x^4\right) \quad \text{for } 0 < x < L/2$$

$$M_{max} = M$$

$$\theta_A = \frac{ML}{6EI}\ ;\ \theta_B = \frac{ML}{3EI}$$

$$EIy = \frac{Mx}{6L}(L - x)(2L - x)$$

$$\delta_{max} = \frac{ML^3}{9\sqrt{3}\,EI} \quad \text{at } x = 0.577L$$

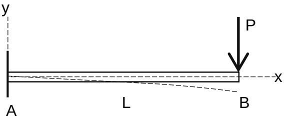


$$M_{max} = M_A = -PL$$

$$\theta_B = \frac{PL^2}{2EI}$$

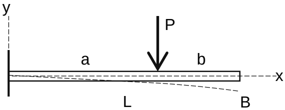
$$\delta_{max} = \delta_B = \frac{PL^3}{3EI}$$

$$EIy = \frac{Px^2}{6}(3L - x)$$



$$M_{max} = M_A = -PL$$

$$\theta_B = \frac{Pa^2}{2EI}$$

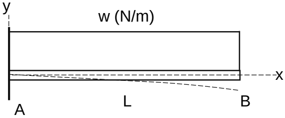


$$\delta_{max} = \delta_B = \frac{Pa^2}{6EI}\,(3L - a)$$

$$EIy = \frac{Px^2}{6}\,(3a - x) \quad \text{for } 0 < x < a$$

$$EIy = \frac{Pa^2}{6}\,(3x - a) \quad \text{for } a < x < L$$

$$M_{max} = M_A = -\,\frac{wL^2}{2}$$

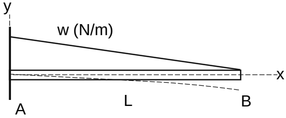


$$\theta_B = \frac{wL^3}{6EI}$$

$$\delta_{max} = \delta_B = \frac{wL^4}{8EI}$$

$$EIy = \frac{wx^2}{24}\left(6L^2 - 4Lx + x^2\right)$$

$$M_{max} = M_A = -\,\frac{wL^2}{6}$$

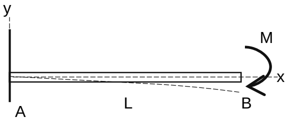


$$\theta_B = \frac{wL^3}{24EI}$$

$$\delta_{max} = \delta_B = \frac{wL^4}{30EI}$$

$$EIy = \frac{wx^2}{120L}\left(10L^3 - 10L^2x + 5Lx^2 - x^3\right)$$

$$M_{max} = M_A = - M$$



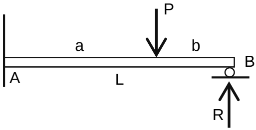
$$\theta_B = \frac{ML}{EI}$$

$$\delta_{max} = \delta_B = \frac{ML^2}{2EI}$$

$$EIy = \frac{Mx^2}{2}$$

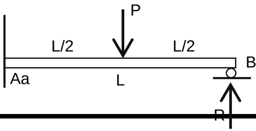
PROPPED BEAM FORMULAS

$$R = \frac{Pa^2\ (3L - a)}{2L^3}$$



$$M_A = - \frac{P}{L^2} \left(b^2 a + \frac{a^2 b}{2}\right)$$

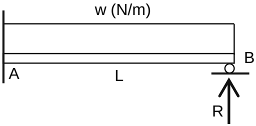
$$R = \frac{5P}{16}$$



$$M_A = - \frac{3PL}{16}$$

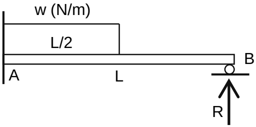
$$R = \frac{3wL}{8}$$

$$M_A = -\frac{wL^2}{8}$$



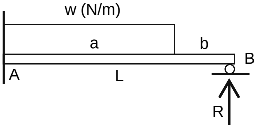
$$R = \frac{7wL}{128}$$

$$M_A = -\frac{9wL^2}{128}$$



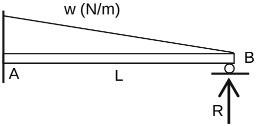
$$R = \frac{wb^3}{8L^3}\,(4L - b)$$

$$M_A = RL - \frac{wa^2}{2}$$



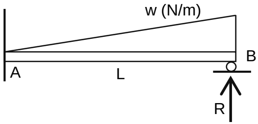
$$R = \frac{wL}{10}$$

$$M_A = -\frac{wL^2}{15}$$



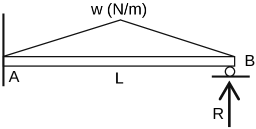
$$R = \frac{11wL}{40}$$

$$M_A = -\frac{7wL^2}{120}$$

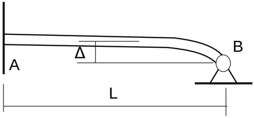


$$R = \frac{11wL}{64}$$

$$M_A = -\frac{5wL^2}{64}$$



$$M_A = \frac{3EI\Delta}{L^2}$$

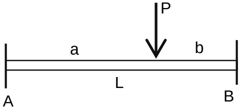


FULLY RESTRAINED BEAM FORMULAS

$$M_A = \frac{Pab^2}{L^2}$$

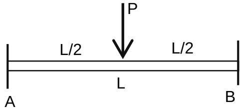
$$M_B = -\frac{Pba^2}{L^2}$$

$$\delta_{mid} = \frac{Pb^2}{48EI}(3L - 4b)$$



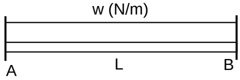
$$M_A = M_B = -\frac{PL}{8}$$

$$\delta_{max} = \frac{PL^3}{192EI}$$



$$M_A = M_B = -\frac{wL^2}{12}$$

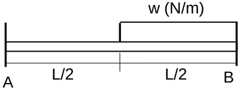
$$\delta_{max} = \frac{wL^4}{384EI}$$



$$M_A = -\frac{5wL^2}{192}$$

$$M_B = -\frac{11wL^2}{192}$$

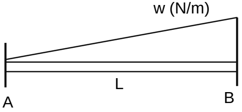
$$\delta_{mid} = \frac{wL^4}{768EI}$$



$$M_A = -\frac{wL^2}{30}$$

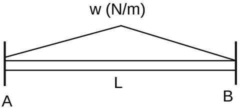
$$M_B = -\frac{wL^2}{20}$$

$$\delta_{mid} = \frac{wL^4}{768EI}$$



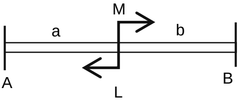
$$M_A = M_B = -\frac{5wL^3}{96}$$

$$\delta_{max} = \frac{7wL^4}{3840EI}$$



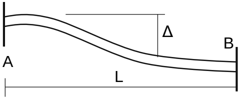
$$M_A = \frac{Mb}{L}\left(\frac{3a}{L} - 1\right)$$

$$M_B = -\frac{Ma}{L}\left(\frac{3b}{L} - 1\right)$$



$$M_A = -\frac{6EI\Delta}{L^2}$$

$$M_B = \frac{6EI\Delta}{L^2}$$

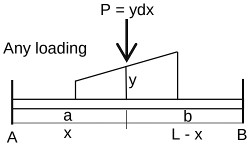


$$M_A = -\int_{x_1}^{x_2} \frac{Pab^2}{L^2}$$

$$M_B = -\int_{x_1}^{x_2} \frac{Pba^2}{L^2}$$

$a = x$; $b = L - x$

$P = y\,dx$



For varying load, $y = f(x)$
For uniform load, $y = w$ (N/m) = constant

INFLUENCE LINES

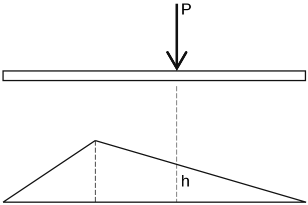
Influence line shows graphically how the movement of a unit load across a structure influences some functions of a structure such as reactions, shears, moments, forces, and deflections.

Influence lines may be defined as a diagram whose ordinates show the magnitude and character of some function of a structure as a unit load moves across the structure. Each ordinate of the diagram gives the values of the function when the load is at that point.

Influence diagram is very useful for moving loads. It is used to determine where to place the loads to cause maximum values of a function and then compute those values.

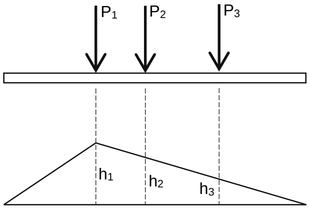
Properties of Influence Line

1. The value of a function due to a single concentrated moving load equals the magnitude of the load multiplied by the ordinate of the influence diagram.



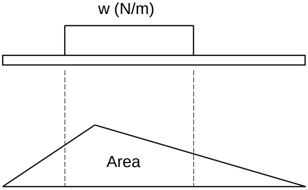
$$\boxed{\text{Function} = P \times h}$$

2. The value of a function due to several concentrated moving loads equals the algebraic sum of the effects of each load described in property number 1



$$\boxed{\text{Function} = P_1\, h_1 + P_2\, h_2 + P_3\, h_3 + \ldots}$$

3. The value of a function due to a uniformly distribute load (w N/m) equals the product of w and the area of the influence line under the uniform load.



$$\boxed{\text{Function} = w \times \text{Area}}$$

APPROXIMATE ANALYSIS OF STRUCTURES

Cantilever Method

Assumptions:

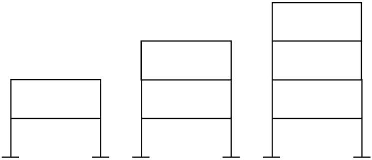
1. A point of inflection occurs at the midspan of each girder.
2. A point of inflection occurs at midheight of each column.
3. The axial force in each column is directly proportional to its distance from the center of gravity of all columns on the level.

Portal Method

Assumptions:

1. The building frame is divided into independent portals.
2. A point of inflection occurs at the midspan of each girder.
3. A point of inflection occurs at the midheight of each column.
4. The horizontal shear at a given story is distributed among the columns such that each interior column resists twice as much as each exterior column.

Note: Portal and Cantilever methods yield the same results for frames such as shown below.



DYNAMIC (IMPACT) LOADING

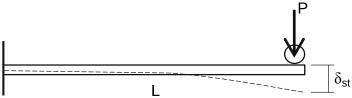
The deformation produced in elastic bodies by impact loads caused them to act as spring, although that is not their designed function.

The spring constant of a beam can be calculated from the following formula:

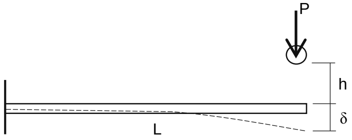
$$\boxed{k = \frac{P}{\delta} \text{ (N/mm or kN/mm)}}$$

where δ is the deformation due to static load P.

Consider the cantilever beam shown.



Static deformation, $\delta_{st} = \frac{PL^3}{3EI}$; $k = \frac{3EI}{L^3}$



If a load P is dropped from a height of h_1 the resulting deformation δ can be computed from:

$$\boxed{\frac{\delta}{\delta_{st}} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}}$$

The maximum stress developed due to impact loading can be determined from the equation.

$$\boxed{\sigma_{max} = \sigma_{st}\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right)}$$

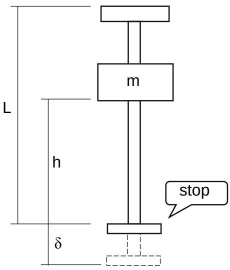
For a mass m dropping freely through a height h before striking a stop at the end of a vertical rod of length L as shown.

$$\delta = \sqrt{\frac{2L}{AE}\ mgh}$$

$$\sigma = \sqrt{\frac{2AE}{AL}\ mgh}$$

$$\sigma = \sqrt{\frac{2E}{AL}\ \frac{mv^2}{2}}$$

where v is the velocity of the mass before impact.



TRUSSES

A truss is a structure composed of slender members joint together at their end points. These are used to support roofs and bridges.

Roof Trusses

Roof trusses are often used as part of a building frame. The roof load is transmitted to the truss at the joints by means of a series of purlins. The roof truss along with its supporting columns is termed as a bent. The space between adjacent bents is called a bay.

Types of Roof Trusses

- Howe Truss
- Pratt Truss
- Fink Truss
- Scissors Truss

- Fan Truss
- Warren Truss
- Bowstring Truss
- Three – Hinged Arc

DETERMINACY OF TRUSSES

In any truss analysis problem, the numbers of unknowns includes the forces in b members and the number of external reaction r, making the number of unknown b + r. Since the truss members are all straight axial force members lying in the same plane, the force system acting at each joint are coplanar and concurrent, making the moment at each joint equal to zero. Thus, in each joint only two equation ($\sum F_x = 0$ & $\sum F_y = 0$) are left to be satisfied. If there is j number of joints, the number of equations that can be made is 2j.

Therefore;

If (b + r) = 2j, the truss is statically determinate

If (b + r) > 2j, the truss is statically indeterminate to the (b + r) - 2j degree

If (b + r) < 2j, the truss is internally unstable

Where:

b = number of bars (or members)
r = number of external reactions (2 for each hinge or pin and 1 for each roller or rocker)
j = number of joints

DEFLECTION OF TRUSSES

The deflection of truss at a joint is given as:

$$\boxed{\text{Deflection, } \delta = \sum \frac{SUL}{AE}}$$

Where S the stress in a member due to actual loads, L is the length of the member, A is the cross – sectional of the member, E is the modulus of elasticity, and U is the stress in the member due to the virtual unit load.

REINFORCED CONCRETE DESIGN

BASIC REQUIREMENTS

DEFINITIONS

The following terms are defined for general use in this chapter. Specialized definitions appear in individual Chapters of Sections.

Admixture – Material other than water, aggregate, or hydraulic cement, used as an ingredient of concrete and added to concrete before or during its mixing to modify its properties.

Aggregate – Granular materials, such as sand, gravel, crushed stone, and iron blast-furnace slag, used with a cementing medium to form a hydraulic cement concrete or mortar.

Aggregate lightweight – Aggregate with a dry, loose weight of 1100 kg/m^3 or less.

Anchorage – In post-tensioning a device used to anchor tendon to concrete member, in pretensioning, a device used to anchor tendon during hardening of concrete.

Bonded tendon – Prestressing tendon that is bonded to concrete either directly or through grouting.

Column – Member with a ratio of height-to-least-lateral dimension of 3 or greater used primarily to support axial compressive load.

Composite concrete flexural members – Concrete flexural members of precast and/or cast-in-place concrete elements constructed in separate placements but so interconnected that all elements respond to loads as a unit.

Concrete – Mixture of Portland cement or any other hydraulic cement, fine aggregate, coarse aggregate, and water, with or without admixture

Concrete, specified compressive strength of, (f'_c) – Compressive strength of concrete used in design expressed in megapascals (MPa). Whenever the quantity f'_c is under a radical sign, square root of numerical value only is intended, and result has units of megapascals (MPa).

Concrete structural lightweight – Concrete containing lightweight aggregate and has an air-dry unit weight not exceeding 1900 kg/m^3. Lightweight concrete without natural sand is termed all-light weight concrete and lightweight concrete in which of the fine aggregate consists of normal weight sand is termed sand-lightweight concrete.

Curvature friction – Friction resulting from bends or curves in the specified prestressing tendon profile.

Deformed reinforcement – Deformed reinforcing bars, bar mats, deformed wire, welded plain wire fabric, and welded deformed wire fabric.

Development length – Length of embedded reinforcement required to develop the design strength of reinforcement at a critical section.

Effective depth of section (d) – Distance measured from extreme compression fiber to centroid of tension reinforcement.

Effective prestress – Stress remaining in prestressing tendons after all losses has occurred, excluding effects of dead load and super imposed load.

Embedment length – Length of embedded reinforcement provided beyond a critical section.

Jacking force – In prestressed concrete, temporary force exerted by the device that introduces tension into prestressing tendons.

Load, dead DL – Dead weight supported by a member.

Load, factored – Load multiplied by appropriate by appropriate load factors, used to proportion members by the strength design method.

Modulus of elasticity – Ratio of normal stress to corresponding strain for tensile of compressive stresses below proportional limit of material.

Modulus, apparent (concrete) – Also known as long-term modulus, is determined by using the stress and strains obtained after the load has been applied for a certain length of time.

Modulus, initial (concrete) – The slope of the stress-strain diagram at the origin of the curve.

Modulus, secant (concrete) – The slope of the line drawn from the origin to a point on the curve somewhere between 25% and 50% of its ultimate compressive strength.

Modulus, tangent (concrete) – The slope of a tangent to the curve at some point along the curve.

Pedestal – Upright compression member with a ratio of unsupported height to average least lateral dimensions of less than 3.

Plain concrete – Concrete that does not conform to definition of reinforced concrete.

Plain reinforcement – Reinforcement that does not conform to definition of deformed reinforcement.

Post-tensioning – Method of prestressing in which tendons are tensioned after concrete has hardened.

Precast concrete – Plain or reinforced concrete element cast elsewhere than its final position in the structure.

Prestressed concrete – Reinforced concrete in which internal stresses have been introduced to reduce potential tensile stresses in concrete resulting from loads.

Pretensioning – Method of prestressing in which tendons are tensioned before concrete is placed.

Reinforced concrete – Concrete reinforced with no less than the minimum amount required by this chapter, prestressed or nonprestressed, and designed on the assumption that the two materials act together in resisting forces.

Spiral reinforcement – Continuously wound reinforcement in the form of a cylindrical helix.

Stirrup – Reinforcement used to resist shear and torsion stresses in a structural member: typically bars, wires or welded wire fabric (smooth or deformed) either single leg or bent into L, U, or rectangular shapes and located perpendicular to or at an angle to longitudinal reinforcement, (The term “stirrups” is usually applied to lateral reinforcement in flexural members and the term “ties” to those in compression members.) See also Tie.

Strength, design – Nominal strength multiplied by a strength reduction factor, Ø

Strength, nominal – Strength of a member or cross-section before application of any strength reduction factors.

Strength, required – Strength of a member or cross-section required to resist factored loads or related internal moments and forces in such combinations.

Tendon – Steel element such as wire, cable, bar, rod, or strand, or a bundle of such elements, used to impart prestresses to concrete.

Tie – Loop of reinforcing bar or wire enclosing longitudinal reinforcement. See also stirrup.

Transfer – Act of transferring stress in prestressing tendons from jacks or pretensioning bed to concrete member.

Wall – Member, usually vertical, used to enclose or separate spaces.

Wobble friction – In prestressed concrete, friction caused by unintended deviation of prestressing sheath or duct from its specified profile.

Yield strength – Specified minimum yield strength or yield point of reinforcement in MPa.

Modulus of Elasticity

Unlike steel and other materials, concrete has no definite modulus of elasticity. Its value is dependent on the characteristics of cement and aggregates used, age of concrete and strengths.
According to NSCP (Section 5.8.5), modulus of elasticity E_c for concrete for values of w_c between 1500 and 2500 kg/m^3 may be taken as

$$\boxed{E_c = w_c^{1.5}\, 0.043 \sqrt{f'_c} \text{ (in MPa)} \qquad \text{Eq. 2 – 1}}$$

Where f'_c is the 28-day compressive strength of concrete in MPa, w_c is the unit weight on concrete in kg/m3. For normal weight concrete, $E_c = 4700 \sqrt{f'_c}$. Modulus of elasticity E_s for nonprestressed reinforcement may be taken as 200, 000 MPa.

Aggregates

Aggregates used in concrete may be fine aggregates (usually sand) and coarse aggregates (usually gravel or crushed stone). Fine aggregates are those that passes through a No.4 sieve (about 6 mm in size). Materials retained are coarse aggregates.

The nominal maximum sizes of coarse aggregate are specified in Section 5.3.3 of NSCP. These are as follows: 1/5 the narrowest dimension between sides of forms, 1/3 the depth of slabs, or ¾ the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, or prestressing tendons or ducts. These limitations may not be applied if, in the judgment of the Engineer, workability and methods of consolidation are such that concrete can be placed without honeycomb or voids.

Water

According to Section 5.3.4, water used in mixing concrete shall be clean and free from injurious amount of oils, acids, alkalis, salts, organic materials, or other substances that may be deleterious to concrete or reinforcement. Mixing water for prestressed concrete or for concrete that will contain aluminum embedments, including that portion of mixing water contributed in the form of free moisture on aggregates shall not contain deleterious amounts of chloride ion. Non-potable (non-drinkable) water shall not be used in concrete unless the following are satisfied: (a) Selection of concrete proportions shall be based on concrete mixes using water from the same source and (b) mortar test cubes made with non-potable mixing water shall have 7-day and 28-day strengths equal to at least 90 percent of strengths of similar specimens made with potable water.

Metal Reinforcement

Metal reinforcement in concrete shall be deformed, except that plain reinforcement be permitted for spirals or tendons; and reinforcement consisting of structural steel, steel pipe, or steel tubing. Reinforcing bars to be welded shall be indicated in the drawings and welding procedure to be used shall be specified. PNS reinforcing bar specifications shall be supplemented to require a report of material properties necessary to conform to welding procedures specified in “Structural Welding Code – Reinforcing Steel” (PNS/AWS D1.4) of the American Welding society and/or “Welding of Reinforcing Bars” (PNS/A5-1554) of the Philippine National Standard.

Deformed Reinforcement

Deformed reinforcing bars shall conform to the standards specified in Section 5.3.5.3 of NSCP. Deformed reinforcing bars with a specified yield strength f_y exceeding 415 MPa shall be permitted, provided f_y shall be the stress corresponding to a strain of 0.35 percent and the bars otherwise conforms to one of the ASTM and PNS specifications listed in Sec. 5.3.5.3.1.

Plain Reinforcement

Plain bars for spiral reinforcement shall conform to the specification listed in Section 5.3.5.3.1 of NSCP. For wire with specified yield strength f_y exceeding 415 MPa, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 415 MPa.

Spacing Limits for Reinforcement

According to Section 5.7.6 of NSCP, the minimum clear spacing between parallel bars in a layer should be d_b but not less than 25 mm. Where parallel reinforcement is placed in two or more layers, bars in the upper layers should be placed directly above bars in the bottom layer with clear distance between layers not less than 25 mm. In spirally reinforced or tied reinforced compression members, clear distance between longitudinal bars shall not be less than $1.5d_b$ nor 40 mm.

In walls and slabs other than concrete joist construction, primary flexural reinforcement shall be spaced not farther apart than three times the wall or slab thickness, or 450 mm.

Bundled Bars

Groups of parallel reinforcing bars bundled in contact to act as a unit shall be limited to four in any one bundle. Bundled bars shall be enclosed within stirrups or ties and bars larger than 32 mm shall not be bundled in beams. The individual bars within a bundle terminated within the span of flexural members should terminate at different points with at least $40d_b$ stagger. Since spacing limitations and minimum concrete cover of most members are based on a single bar diameter d_b, bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

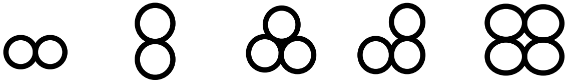
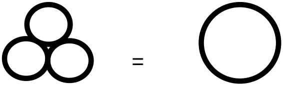


Figure 2 – 1: Bundled-bar arrangement

Diameter of single bar equivalent to bundled bars according to NSCP to be used for spacing limitation and concrete cover.



3 – 25 mm equivalent diameter, D

$$\frac{\pi}{4}(25)^2 \times 3 = \frac{\pi}{4}(D)^2 \text{ ; } D = 43.3 \text{ mm}$$

Figure 2 – 2: Equivalent single bar

Concrete Protection for Reinforcement

Steel reinforcement in concrete should be provided with adequate covering as provided in Section 5.7.7 of NSCP. These covering depend on the type of exposure of the member and fire protection. Some of these values are; for concrete cast and permanently exposed to earth such as footings, the minimum concrete cover is 75 mm. For concrete members exposed to weather, 40 to 50 mm. For concrete not exposed to weather or in contact with ground, the minimum cover is 20 mm for slabs, walls, and joists, and 40 mm for beams and columns.

Bundled Bars

For bundled bars, the minimum concrete cover shall be equal to the equivalent diameter of the bundle, but need not be greater than 50 mm, except for concrete cast against and permanently exposed to earth, the minimum cover shall be 75 mm.

Standard Hooks

The term standard hook refers to one of the following:

(a) 180° bend plus $4d_b$ extension but not less than 65 mm at free end,

(b) 90° bend plus $12d_b$ extension, at free end of bar,

(c) For stirrups and tie hooks:

(1) 16 mm bar and smaller, 90° bend plus $6d_b$ extension at free end of bar, or

(2) 20 mm bar and 25 mm bar, 90° bend plus $12d_b$ extension at free end of bar, or

(3) 25 mm bar and smaller, 135° bend plus $6d_b$ extension at free end of bar.

Cast-in-place Concrete (nonprestressed). The following minimum concrete cover shall be provided for reinforcement:

		Minimum cover, mm
(a)	Concrete east against and permanently exposed to earth	75
(b)	Concrete exposed to earth or weather: 20 mm through 36 mm bars 16 mm bar, W31 or D31 wire, and smaller	 50 40
(c)	Concrete not exposed to weather or in contact with ground:	

	Slabs, walls, joists 32 mm bar and smaller Beams, columns Primary reinforcement, ties, stirrups, spirals Shells, folded place members: 20 mm bar and larger 16 mm bar, W31 or D31 wire, and smaller	 20 40 20 15

Precast concrete (Manufactured Under Plant Conditions). The following minimum concrete shall be provided for reinforcement.

		Minimum cover, mm
(a)	Concrete exposed to earth or weather: Wall panels: 32 mm bar or smaller Other members: 20 mm through 32 mm bars 16 mm bar, W31 or D31 wire, and smaller	 20 40 30
(b)	Concrete not exposed to weather or in contact with ground Slabs, walls, joists:	 15

	32 bar and smaller Beams, columns: Primary reinforcement Ties, stirrups, spirals Shells, folded plate members: 20 mm bar and larger 16 mm bar, W31 or D31 wire, and smaller	 d_b but not less than 15 & need not exceed 40 10 15 10

Minimum Bend Diameter

The diameter of bend measured on the inside of the bar, other than for stirrups and ties in sizes 10 mm through 15 mm shall not be less than the following: (a) $6d_b$ for 10 mm or 25 mm bar, (b) $8d_b$ for 28 mm to 32 m bar, and (c) $10d_b$ for 36 mm bar.

The inside diameter of bend of stirrups and ties shall not be less than $4d_b$ for 16 mm bar and smaller. For bars larger than 16 mm, the diameter of bend shall be in accordance with the previous paragraph.

Storage Materials

Cement and aggregates shall be stored in such manner as to prevent deterioration or intrusion of foreign matter. Any material that has deteriorated or has been contaminated shall not be used for concrete.

Concrete Proportions

Proportions of materials for concrete shall be established to provide: (a) workability and consistency to permit concrete to be worked readily into forms and around reinforcement under conditions of placement to be employed, without segregation or excessive bleeding, (b) resistance to special exposures, and (c) conformance with strength test requirements.

Where different materials are to be used for different portions of proposed work, each combination shall be evaluated. Concrete proportions, including water-cement ratio, shall be established based on field experience and/or trial mixtures with materials to be employed.

Loads

The most important and most critical task of an engineer is the determination of the loads that can be applied to a structure during its life, and the worst possible combination of these loads that might occur simultaneously. Loads on a structure may be classified as dead loads or live loads.

Dead Load

Dead loads are loads of constant magnitude that remain in one position. This consists mainly of the weight of the structure and other permanent attachments to the frame.

Live Load

Live loads are loads that may change in magnitude and position. Live loads that move under their own power are called moving loads. Other live loads are those caused by wind, rain, earthquakes, soils, and temperature changes. Wind and earthquake loads are called lateral loads.

Arrangement of Live Load

Live loads may be applied only to the floor or roof under consideration, and the far ends of columns built integrally with the structure may be considered fixed. It is permitted by the code to assume the following arrangement of live loads: (a) Factored dead load on all spans with full factored live load on two adjacent spans, and (b) Factored dead load on all spans with full factored live load on alternate spans.

Table 2 – 1: Uniform and Concentrated Loads (NSCP)

<table>
<tr><td colspan="3">Use of occupancy</td><td>Uniform Load, Pa</td><td>Concentrated Load, N</td></tr>
<tr><td colspan="2">Category</td><td>Description</td><td></td><td></td></tr>
<tr><td>1</td><td>Armories</td><td></td><td>7200</td><td>0</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">Assembly areas and auditorium</td><td>Fixed seating areas</td><td>2400</td><td>0</td></tr>
<tr><td>Movable</td><td>4800</td><td>0</td></tr>
</table>

	and balconies therewith	seating and other areas		
		Stage areas and enclosed platforms	6000	0
3	Cornices, marquees & residential balconies		3000	0
4	Exit facilities		4800	0
5	Garages	General storage and/or repair	4800	
		Private pleasure car storage	2400	

Use of occupancy			Uniform load Pa	Concentrated load, N
Category		Description		
6	Hospitals	Wards and rooms	2000	4500
7	Libraries	Reading rooms	3000	4500
		Stock rooms	6000	6700
8	Manufacturing	Light	3600	8900
		Heavy	6000	13400

9	Offices		2400	8900
10	Printing plants	Press rooms	7200	11200
		Composing and linotype rooms	4800	8900
11	Residential		2000	0
12	Rest rooms	Not less than the load for the occupancy with which they are associated but need not exceed 2400 Pa		
13	Reviewing stands, grandstands and bleachers		4800	0
14	Roof deck	Same as area served for the type of occupancy		
15	Schools	Classrooms	2000	4500
16	Sidewalks and	Public	12000	

	driveways	access		
17	Storage	Light	6000	
		Heavy	12000	
18	Stores	Retail	3600	8900
		Wholesale	4800	13400
19	Low cost housing unit		1500	0

Table 2 – 2: Minimum Roof Live Loads (NCSP)

Roof slope	Tributary Loaded Area for structural Member		
	0 to 20 m^2	21 to 60 m^2	Over 60 m^2
1. Flat or rise less than 1 vertical to 3 horizontal; arch or dome with rise less 1/8 of span	1000 Pa	800 Pa	600 Pa
2. Rise 1 vertical per 3 horizontal to less than 1 horizontal; Arch or dome with rise 1/8 of span to less than 3/8 of span or greater.	800 Pa	700 Pa	600 Pa
3. Rise 1 vertical	600 Pa	600 Pa	600 Pa

to 1 horizontal; Arch or dome with rise 3/8 of span or greater.			
4. Awnings, except cloth covered	250 Pa	250 Pa	250 Pa
5. Green houses, lathhouses and agricultural buildings	500 Pa	500 Pa	500 Pa

Use			Vertical load Pa	Lateral Load Pa
Category		Description		
1	Construction, public access at te site (Live load)	Walkway, Canopy	7200	
2	Grandstands, reviewing stands and bleachers (live load)	Seats and footboards	1750	
3	Stage accessories	Gridirons and fly galleries	3600	
		Loft block wells	3650	3650

		Head block wells and sheave beams	3650	3650
4	Ceiling framing	Over stages	1000	
		All uses except over stages	500	
5	Partitions and interior walls			250
6	Elevators and dumbwaiters (Dead loan and Live load)		2 by total loads	
7	Mechanical and electrical equipment		Total load	
8	Cranes (Dead and live load)	Total load including impact increase	1.25 by total load	0.10 by total load
9	Balcony railings, guardrails and handrails	Exit facilities serving an occupant load greater than 50		750
		other		300
10	Storage racks	Over 2.4 m	Total	

			loads	

Refer to Chapter 2 of NSCP

Load Factors

Dead load, DL………………………….. 1.4
Live load, LL …………………..……….. 1.7
Wind load ……………………..………. 1.7
Earthquake, E ………………………….. 1.87
Earth or water pressure, H…………… 1.7

Required Strength (Factored load), U

Structure and structural members should be designed to have design strengths at all sections at least equal to required strengths calculated for the factored loads and forces in any combination of loads.

- ❖ The required strength U to resist dead load DL and live load LL is

$$\boxed{U = 1.4DL + 1.7LL \qquad \text{Eq. 2-2}}$$

- ❖ If resistance to structural effects of a specified wind load W are included in the design where load combinations includes both full value and zero value of LL to determinate the more severe condition,

$U = 0.75(1.4DL + 1.7LL + 1.7W)$ Eq. 2-3

And $U = 0.9DL + 1.3W$ Eq. 2-4

But not less than 1.4DL + 1.77LL Eq. 2-5

- ❖ If resistance to specified earthquake loads or forces E are included in the design

U = 0.75 (1.4DL + 1.7LL + 1.87E) Eq. 2 – 6

And U = 0.9DL + 1.43E Eq. 2 – 7

but not less than 1.4DL + 1.7LL Eq. 2 – 8

- ❖ if resistance to earth pressure H is include in design

U = 1.4DL + 1.7LL + 1.7H Eq. 2 – 9

Where DL or LL reduce the effect of H

U = 0.90 DL Eq. 2 – 10

but not less than 1.4DL + 1.7LL

- ❖ If resistance to loadings due to weight and pressure of fluids with well-defined densities and controllable maximum heights F is included in design, such loading shall have a load factor of 1.4 and be added to all loading combinations that include live load;

- ❖ If resistance to impact effects is taken into account in design, such effects shall be included with live load LL.
- ❖ Where structural effects T of differential settlement, creep, shrinkage, or temperature change are significant in design

$$U = 0.75\ (1.4DL + 1.4T + 1.7LL) \qquad \text{Eq. 2 – 11}$$

but required strength U shall not be less than

$$U = 1.4(DL + T) \qquad \text{Eq. 2 – 12}$$

<u>Strength Reduction Factors, φ (phi)</u>

The design strength provided by a concrete member, its connections to other members, and its cross sections, in term of flexure, axial load, shear, and torsion shall be taken as the nominal strength multiplied by a strength reduction factor Φ having the following values:

(a) Flexure without axial load ……………….................. 0.90
(b) Axial tension, and axial tension with flexure ……….. 0.90
(c) Axial compression, and axial compression with
flexure:
(1) Spiral reinforcement …………………………. 0.75
(2) Tie reinforcement ……………………………. 0.70
(d) Shear and torsion …………………………………….. 0.85

(e) Bearing on concrete …………………………………….. 0.70

ANALYSIS AND DESIGN OF BEAMS

Notations and Symbols Used in the Book

a = depth of equivalent stress block, mm

A_s = area of tension reinforcement, mm2

A_{sk} = area of skin reinforcement per unit height in one side face, mm2/m

b = width of compression face of member, mm

c = distance from extreme compression fiber to neutral axis, mm

d = distance from extreme compression fiber to centroid of tension reinforcement, mm

d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire, mm

E_c = modulus of elasticity of concrete, MPa

E_s = modulus of elasticity of steel = 200, 000 MPa

f'_c = specified compressive stress of concrete, MPa

f_s = calculated stress in reinforcement at service loads, MPa

f_y = specified yield strength of steel, MPa

h = overall thickness of member, mm

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross-section

M_n = nominal moment, N-mm

M_u = factored moment at section, N-mm

B_1 = factor defined in Section 5.10.2.7.3

ε_c = strain in concrete (maximum = 0.003)

ε_s = strain in steel below yield point = f_s/ε_s
ε_y = strain in steel at yield point = f_y/ε_s
ρ = ratio of tension reinforcement = A_s/bd
ρ_b = balance steel ratio (See Section 5.10.3.2)
Ø = strength reduction factor (See Sec. 5.9.3)

Balanced Design

A design so proportioned that the maximum stresses in concrete (with strain of 0.003) and steel (with strain of f_y/ε_s) are reached simultaneously once the ultimate load is reached, causing them to fail simultaneously.

Underreinforced Design

A design in which the steel reinforcement is lesser than what is required for balance condition. If the ultimate load is approached, the steel will begin to yield although the compression concrete is still understressed. If the load is further increased, the steel will continue to elongate, resulting in appreciable deflections and large visible cracks in the tensile concrete. Failure under this condition is ductile and will give warning to the user of the structure to decrease the load.

Overreinforced Design

A design in which the steel reinforcement is more than what is required for balance condition. If the beam is overreinforced, the steel will not yield before failure. As the load is increased, deflections are not noticeable although the compression concrete is highly stressed, and failure occurs suddenly without warning to the user of the structure. Overreinforced as well as balanced design should be avoided in concrete because of its brittle property, that is why the Code limits the tensile steel percentage (ρ_{max} = 0.75ρ_b) to ensure underreinforced beam with ductile type of failure to give occupants warning before failure occurs.

Assumptions in Strength Design in Flexure
(Code Sections 5.10.2)

1. Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis. Except for deep flexural members with overall depth to clear span ratio, h/L > 2/5 for continuous spans and h/L > 4/5 for simple spans, a nonlinear distribution of strain shall be considered (See Sec. 5.10.7)
2. Maximum usable strain at extreme concrete compression fiber, ε_c shall be assumed equal to 0.003.
3. For fs below f_y, f_s shall be taken as E_s x ε_s. For $\varepsilon_s > \varepsilon_y$, $f_s = f_y$.
4. Tensile strength of concrete shall be neglected in axial and flexural calculations.
5. Relationship between compressive stress distribution and concrete strain may be assumed rectangular, trapezoidal, parabolic, or any other form that results in prediction of strength in substantial agreement with results of compressive tests.
6. For rectangular distribution of stress:
   (a) Concrete stress of 0.85f'_c shall be assumed

uniformly distributed over an equivalent compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance of $a = \beta_1 c$ from the fiber of maximum compressive strain.

(b) Distance c from fiber of maximum strain to the neutral axis shall be measured in the direction perpendicular to N.A.

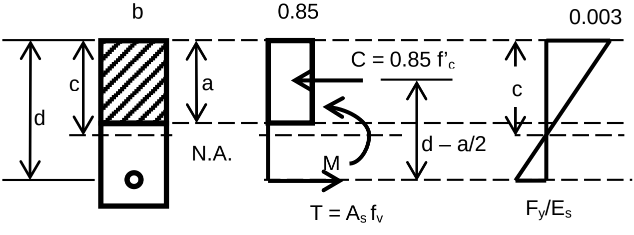
(c) Factor β_1 shall be taken as 0.85 for $f'_c \leq 30$ MPa and β_1 shall be reduced continuously at a rate of 0.008 for each 1 MPa of strength in excess of 30 MPa, but β_1 shall not be taken less than 0.65. i.e.

(i) For $f'_c \leq 30$ MPa, $\beta_1 = 0.85$

(ii) For $f'_c > 30$ MPa,

$\beta_1 = 0.85 - 0.008(f'_c - 30)$ but shall not be less than 0.65

SINGLY REINFORCED BEAM



Stress Diagram   Strain Diagram

$$a = \beta_1 c \qquad \text{Eq. 2 - 13}$$

For $f'_c \leq 30$ Mpa, $\beta_1 = 0.85$

For $f'_c > 30$ Mpa, $\beta_1 = 0.85 - 0.008\,(f'_c - 30)$

but shall not be less than 0.65

$[\Sigma F_H = 0]$ C = T

$$0.85\, f'_c\, a\, b = A_s\, f_v$$

$$a = \frac{A_s\, f_y}{0.85\, f'_c b} \qquad \text{Eq. 2 – 14}$$

Multiplying both sides by d/d:

$$a = \frac{A_s\, f_y}{0.85\, f'_c b} \times \frac{d}{d} = \frac{A_s}{bd}\ \frac{f_y\, d}{0.85\, f'_c}$$

The term $\frac{A_s}{bd}$ is called the ratio of steel reinforcement and is denoted as ρ

$$\rho = \frac{A_s}{bd} \qquad \text{Eq. 2 – 15}$$

$$\text{And } a = \frac{\rho\, f_y\, d}{0.85\, f'_c} \qquad \text{Eq. 2 – 16}$$

$$\omega = \frac{\rho\, f_y}{f'_c} \qquad \text{Eq. 2 – 17}$$

$$a = \frac{\omega\, d}{0.85} \qquad \text{Eq. 2 – 18}$$

Nominal Moment Capacity:

From the stress diagram in the figure above:

$$M_n = C \times \left(d - \frac{a}{2}\right) = 0.85\, f'_c\, ab\, (d - \tfrac{1}{2} a)$$

$$M_n = 0.85\, f'_c\, \frac{\omega\, d}{0.85} = b\, (d - \frac{1}{2}\frac{\omega\, d}{0.85})$$

$$M_n = f'_c\ \omega\, b\, d^2 = (1 - 0.59\omega) \qquad \text{Eq. 2 – 19}$$

Ultimate Moment Capacity:

$M_u = \Phi\, M_n$ (where $\Phi = 0.90$ for flexure)

$$M_u = \Phi\, f'_c\ \omega\, b\, d^2 = (1 - 0.59\omega) \qquad \text{Eq. 2 – 20}$$

Coefficient of Resistance

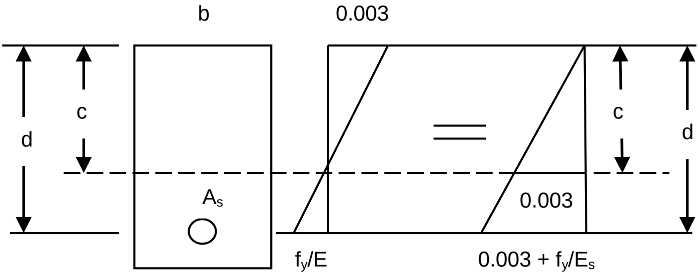
$$R_u = f'_c\ \omega\ = (1 - 0.59\omega) \qquad \text{Eq. 2 – 21}$$

$$M_u = \Phi\, R_u\ b\, d^2 \qquad \text{Eq. 2 – 22}$$

Solving for ω and replacing it with $\frac{\rho\, f_y}{f'_c}$, yields the following formula for the steel ratio ρ:

$$\rho = \frac{0.85\, f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2\, R_u}{0.85\, f'_c}}\right] \qquad \text{Eq. 2 – 23}$$

Balanced Steel Ratio, ρ_b



Strain diagram

By ratio and proportion:

$$\frac{c}{d} = \frac{0.003}{0.003 + f_y/E_s} \;;\; E_s = 200,000$$

$$c = \frac{0.003}{0.003 + \frac{f_y}{200,000}}\, d = \frac{600}{600 + f_y}\, d$$

But $a = \beta_1 c$;

$$c = \frac{a}{\beta_1} = \frac{\frac{\rho f_y d}{0.85 f'_c}}{\beta_1} = \frac{\rho f_y d}{0.85 f'_c \beta_1}$$

$$\frac{\rho f_y d}{0.85 f'_c \beta_1} = \frac{600}{600 + f_y}\, d$$

$$\boxed{\rho_b = \frac{0.85 f'_c \beta_1 600}{f_y (600 + f_y)} \qquad \text{Eq. 2 – 24}}$$

Maximum and Minimum Steel Ratio

Section 5.10.3.3: For flexural members the ratio of reinforcement ρ provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions.

$$\boxed{\rho_{max} = 0.75\ \rho_b \qquad \text{Eq. 2 – 25}}$$

This limitation is to ensure that the steel reinforcement will yield first to ensure ductile failure.

Section 5.10.5.1: At any section of a flexural member where positive reinforcement is required by analysis, the ratio ρ provided shall not be less than given by $1.4/f_y$

$$\rho_{min} = \frac{1.4}{f_y} \qquad \text{Eq. 2 – 26}$$

The provision for minimum amount of reinforcement applies to beams, which for architectural and other reasons are much larger in cross-section than required by strength as a reinforced concrete section becomes less than that of the corresponding plain concrete section computed from its modulus of rupture. Failure in such a case can be quite sudden.

STEPS IN DESIGNING A SINGLY REINFORCED RECTANGULAR BEAM FOR FLEXURE:

Note: The assumptions made in steps II, V, VIII are the author's recommendation based on his experience.

I. Identify the values of the dead load and live load to be carried by the beam. (DL & LL)
II. Approximate the weight of beam (DL) between 20% to 25% of (DL + LL). This weight is added to the dead load.
III. Compute the factored load and factored moment:

ex., Factored Load = 1.4DL + 1.7 LL

IV. Compute the factored moment to be resisted by the beam, Mu
V. Try a value of steel ratio ρ from $0.5\rho_b$ to $0.6\rho_b$, but must not be less than ρ_{min}. This value of ρ due to rounding-off of the number of bars to be used, for it not to exceed the maximum ρ of $0.75\rho_b$.

$$\rho_b = \frac{0.85\, f'_c\, \beta_1\, 600}{f_y(600 + f_y)}$$

$\beta_1 = 0.85$ $f'_c \leq 30$ Mpa
$\beta_1 = 0.85 - 0.008\,(f'_c - 30)$ $f'_c > 30$ Mpa
$\rho_{min} = 1.4/f_y$

VI. Compute the value of ω, $\omega = \frac{\rho\, f_y}{f'_c}$
VII. Solve for bd^2:

$$M_u = \Phi\, f'_c\, \omega\, b\, d^2\, (1 - 0.59\omega)$$
$$bd^2 =$$

VIII. Try a ratio d/b (from d = 1.5 b to d = 2b), and solve for d.

(round-off this value to reasonable dimension)

Check also the minimum thickness in beam required by the Code as given in Table 2 – 4 of Page 103.

After solving for d, substitute its value to Step VII, and solve for b.

Compute the weight of the beam and compare it to the assumption made in Step II.

IX. Solve for the required steel area and number of bars.

$A_s = \rho\ b\ d$

Number of bars (diameter = D)

$\frac{\pi}{4}\ (D)^2$, number of bars = A_s

<u>STEPS IN COMPUTING THE REQUIRED TENSION STEEL AREA A_S OF A BEAM WITH KNOWN MOMENT M_U AND OTHER BEAM PROPERTIES:</u>

I. Solve for ρ_{max} and $M_{u\ max}$

$$\rho_{max} = 0.75\ \rho_b$$

$$\rho_{max} = \frac{0.85\ f'_c\ \beta_1\ (600)}{f_y\ (600+ f_y)} = \rho$$

$$\omega = \rho \frac{f_y}{f_c} =$$

$$M_{u\,max} = \Phi\, f'_c\ \omega\, b\, d^2\, (1 - 0.59\omega)$$

If $M_u \leq M_{u\,max}$ design as Singly Reinforced (Step II)
If $M_u > M_{u\,max}$ design as Doubly Reinforced (Step III)

II. Solve for ρ:

$$M_u = \Phi\, R_u\ b\, d^2\ ;\ R_u =$$

$$\rho = \frac{0.85\, f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2\, R_u}{0.85\, f'_c}}\right] =$$

$$\mathbf{A_s = \rho\, b\, d =}$$

III. Compression reinforcement is needed.

STEPS IN COMPUTING M_U OF A BEAM WITH KNOWN TENSION STEEL AREA A_S AND OTHER BEAM PROPERTIES:

I. Solve for ρ; $\rho = \frac{A_S}{bd}$

II. Check if steel yields by computing ρ_b;

$$\rho_b = \frac{0.85\, f'_c\, \beta_1\, (600)}{f_y\, (600 + f_y)}$$

If $\rho \leq \rho_b$ steel yields, proceed to Step III
If $\rho > \rho_b$ steel does not yield, proceed to Step IV.

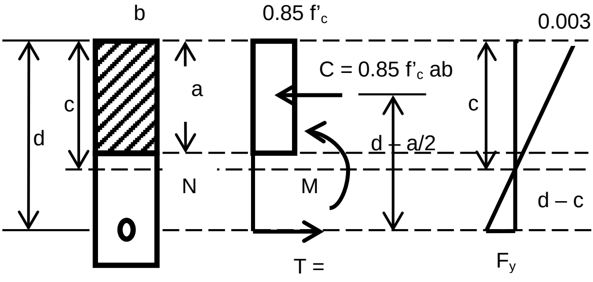
Note: If $\rho \leq \rho_{min}$ the given A_s is not adequate for the beam dimension.

III. $\rho \leq \rho_b$

$$\omega = \frac{\rho f_y}{f'_c}$$

$$\mathbf{M_u = \Phi f'_c \ \omega b d^2 (1 - 0.59\omega) =}$$

IV. $\rho > \rho_b$



Solve for f_s form the strain diagram:

$\frac{f_s/E_s}{d - c} = \frac{0.003}{c}$; $f_s = 600 \frac{d - c}{c}$

$[\Sigma F_H = 0]$ T = C

$$A_s f_v = 0.85 f'_c a b; a = \beta_1 c$$

$$A_s 600 \frac{d - c}{c} = 0.85 f'_c (\beta_1 c) b$$

$$600 A_s (d - c) = 0.85 \beta_1 f'_c b c^2$$

Solve c by quadratic formula and solve for f_s and a:

$f_s = 600 \frac{d - c}{c}$; $a = \beta_1 c$

$\mathbf{M_u = \Phi T (d - a/2) = \Phi A_s f_s (d - a/2)}$

or

$\mathbf{M_u = \Phi C (d - a/2) = \Phi 0.85 f'_c a b (d - a/2)}$

Minimum Thickness of Flexural Members

According to Section 5.9.5 of NSCP, minimum thickness stipulated in Table 2 – 4 shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

	Minimum thickness, h			
	Simply	One end	Both ends	Cantilever

	supported	continuous	continuous	
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	L/20	L/24	L/28	L/10
Beams or ribbed one-way slabs	L/16	L/18.5	L/21	L/8

Span length L is in millimeters

Values given shall be used directly for members with normal density concrete (w_c = 2300 kg/m^3) and grade 415 reinforcement. For other conditions, the values shall be modified as follows:

(a) For structural lightweight concrete having unit weights in the range 1500 – 2000 kg/m3, the values shall be multiplied by (1.65 – 0.0005 w_c) but not less than 1.09, where w_c is the unit mass in kg/m^3.

(b) For f_y other than 415 MPa, the values shall be multiplied by (0.4 + f_y/700).

DOUBLY REINFORCED BEAM

Occasionally, beams are restricted in small sizes by space or aesthetic requirements to such extent that the compression concrete should be reinforced with steel to carry compression. Compression reinforcement is needed to increase the moment capacity of a beam beyond that of a tensilely reinforced beam with a maximum steel percentage of $0.75\rho_b$. Aside from these reactions, compression reinforcement makes beams tough and ductile and reduces long-time deflection of beams.

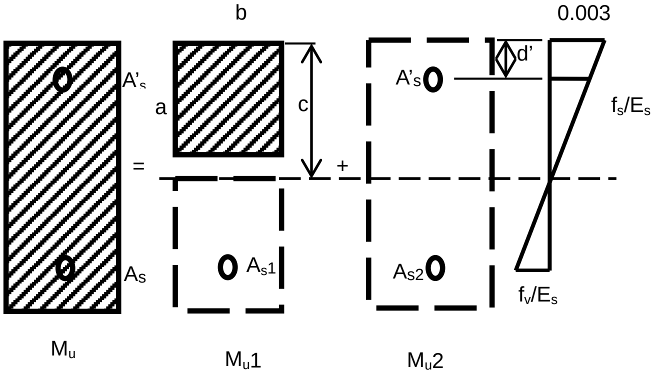
Compression steel also helps the beam withstand stress reversals that might occur during earthquakes. Continuous compression bars are also helpful for positioning stirrups and keeping them in place during concrete placement and vibration. Various tests show that compression reinforcement also prevents the beam to collapse even if the compression concrete crushes especially if it is enclosed by stirrups.

According to Section 5.7.10 of NSCP, compression steel in beams must be enclosed by lateral ties, at least 10 mm in size for longitudinal bars 32 mm or smaller, and at least 12 mm in size for 36 mm and bundled bars. Deformed wire or welded wire fabric of equivalent area is allowed. The spacing of these ties shall not exceed 16 longitudinal bar diameters, 48 tie bar or wire diameters, or least dimension of the compression member.

Analysis of Doubly Reinforced Beam

Doubly reinforced beam is analyzed by dividing the beam into two couples, Mu1 and Mu2 as shown in the figure. M_{u1} is the couple due to compression concrete and the part of the tension steel A_{s1}, and M_{u2} is the couple due to the compression steel A'_s

and the other part of the tension steel area A_{s2}.



Compression reinforcement is provided to ensure ductile failure (i.e. tension steel must yield). For this reason, therefore, the stress in tension steel (A_s) is always to f_y. On the other hand, the stress of compression steel (A'_s) may either be f_y or below f_v. This stress must always be checked.

If the compression steel yields, then $A'_s = A_{s2}$, otherwise $A'_s = A_{s2}$ f_y/f'_s where f'_s is the stress of compression steel and is given by the following equation

$$f_s = 600 \frac{c - d'}{c} \qquad \text{Eq. 2 - 27}$$

According to Section 5.10.3.3 of NSCP, for members with compression reinforcement, the portion of ρ_b equalized by compression reinforcement need not be reduced by the 0.75 factor. Thus, the maximum permissible A_s is:

$$A_{s\,max} = 0.75\,\rho_b\,b\,d + A'_s\,\frac{f'_s}{f_y} \qquad \text{Eq. 2 - 28}$$

The expression $0.75\,\rho_b\,b\,d = A_{s1}$, and $A'_s\,\frac{f'_s}{f_y} = A_{s2}$.

STEPS IN COMPUTING A_S AND A'_S FOR DOUBLY REINFROCED BEAM, GIVEN M_U AND OTHER BEAM PROPERTIES

I. Solve for ρ_{max} and $M_{u\,max}$

$$\rho_{max} = 0.75\,\rho_b$$

$$\rho_{max} = \frac{0.85\,f'_c\,\beta_1\,600}{f_y\,(600 + f_y)} = \rho$$

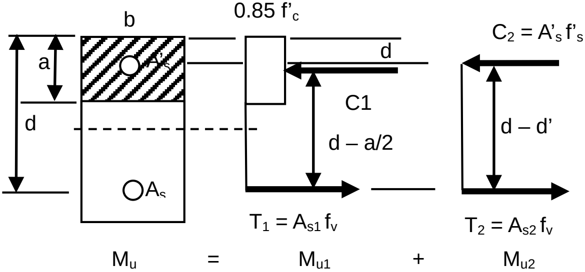
$$\omega = \rho\,\frac{f_y}{f_c} =$$

$$M_{u\,max} = \phi\,f'_c\,\omega\,b\,d^2\,(1 - 0.59\omega)$$

If $M_u \leq M_{u\,max}$ design as Singly Reinforced
(See Page 101)

If $M_u > M_{u\,max}$ design as Doubly Reinforced (Step II)

II. Solve for A_{s1}: $A_{s1} = \rho_{max}\,b\,d$



III. Solve for a and c:

$[C_1 = T_1]$ $0.85 f'_c\, a\, b = A_{s1} f_y$: a =

$a = \beta_1 c$: c =

IV. Solve for M_{u1}, M_{u2} and A_{s2}

$M_{u1} = M_{u\,max}$

$M_{u1} = M_u - M_{u1}$

$M_{u1} = \Phi\, T_2\,(d - d') = \Phi\, A_{s2}\, f_y\,(d - d_1)$

A_{s2} =

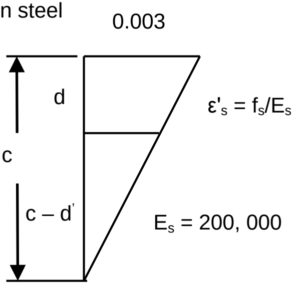
V. Solve for the stress of compression steel

$$\frac{f_s/E_s}{d - c} = \frac{0.003}{c}$$

$$\boxed{f_s = 600\,\frac{d - c}{c}}$$

If $f_s \geq f_y$ proceed to Step VI

If $f_s < f_y$ proceed to Step VII



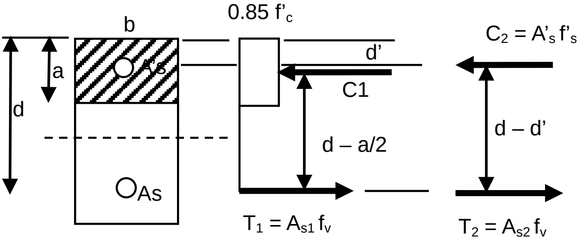
VI. If $f'_s \geq f_y$ then use $f'_s = f_y$ (compression steel yields)

$A'_s = A_{s2}$

VII. If $f'_s < f_y$ then use f'_s (compression steel will not yields)

$A'_s = A_{s2}\, f_y/f'_s$

<u>STEPS IN COMPUTING M_U OF A DOUBLY REINFORCED BEAM WITH A GIVEN A_S, A'_S, AND OTHER BEAM PROPERTIES</u>



$M_u \quad = \quad M_{u1} \quad + \quad M_{u2}$

I. Assume compression steel yields ($f'_s = f_y$)

$A_{s2} = A'_s =$

$A_{s1} = A_s - A_{s2} =$

II. Solve for a and c:

$[C_1 = T_1]$ $0.85\, f'_c\, a\, b = A_{s1}\, f_y$: $a =$

$a = \beta_1\, c$: $c =$

III. Solve for the stress in compression steel

$$f'_s = 600\ \frac{c - d'}{c}$$

If $f'_s \geq f_y$, proceed to step IV

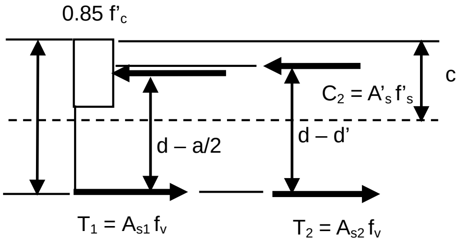
If $f'_s < f_y$, proceed to step V

IV. Since $f'_s \geq f_y$, compression steel yields

$$M_u = M_{u1} + M_{u2} = \Phi\ T_1\ (d - a/2) + \Phi\ T_2\ (d - d')$$

$$\mathbf{M_u = \Phi\ A_{s1}\ f_y\ (d - a/2) + \Phi\ A_{s2}\ f_y\ (d - d')}$$

V. If $f'_s < f_y$ assumption is wrong, compression steel does not yield



$$f'_s = 600\ \frac{c - d'}{c}$$

From the stress diagram:

$$[C_1 + C_2 = T]$$

$$0.85\ f'_c\ a\ b = A'_s\ f'_s = A_s\ f_y$$

$$0.85\ f'_c\ \beta_1\ c\ b = A'_s + f'_s = 600\ \frac{c - d'}{c} = A_s\ f_y$$

Solve for c by quadratic formula

Solve for f'_s, $f'_s = 600\ \frac{c - d'}{c} =$

Solve for a, $a = \beta_1\ c$, c =

Solve for M_u:

$$M_u = M_{u1} + M_{u2} = \Phi\ C_1\ (d - a/2) + \Phi\ C_2\ (d - d')$$

$$\mathbf{M_u = \Phi\ 0.85\ f'_c\ a\ b\ (d - a/2) + \Phi\ A'_s\ f'_s\ (d - d')}$$

DEEP BEAMS

According to Section 5.10.7.1 of the Code, beams with overall depth to clear span ratios greater than 2/5 for continuous spans, or 4/5 for simple spans, shall be designed as deep flexural members taking into account nonlinear distribution of strain and lateral buckling.

Beams with web depth that exceed 900 mm have a tendency to develop excessive wide cracks in the upper parts of their tension zones. According to Section 5.10.6.7 of NSCP, if the depth of a web exceeds 900 mm, longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member for a distance d/2 nearest the flexural tension reinforcement. The area of skin reinforcement A_{sk} per meter of height on each side face shall be

$$A_{sk} \geq 1.016\ (d - 750)$$

The maximum spacing of the skin reinforcement shall not exceed the lesser of d/b and 300 mm. Such reinforcement may be included in strength computations if a strain compatibility analysis is made to determine stresses in the individual bars or wires. The total area of longitudinal skin reinforcement in both faces need not exceed one-half of the required flexural tensile reinforcement.

T-Beams

Reinforced concrete floors usually consist of slabs and beams, which are placed or poured monolithically. In this effect, the beam will have an extra width at the top (which is usually under compression) called flanges, and the resulting section is called a T-beam. The beam may also be L-shaped if it is located at the end of a slab.

Analysis and Design of T-Beams

The compression block of a T-beam can fall either within the flange only or partly in the web. If it falls within the flange as shown in Figure (a), the rectangular beam formulas (in chapter 2) apply since the concrete below neutral axis is assumed to be cracked and its shape has no effect on the flexure calculations (other than weight). If however it covers the web as shown in Figure (b), the compression concrete no longer consist of a single rectangle and thus the rectangular beam formulas do not apply.

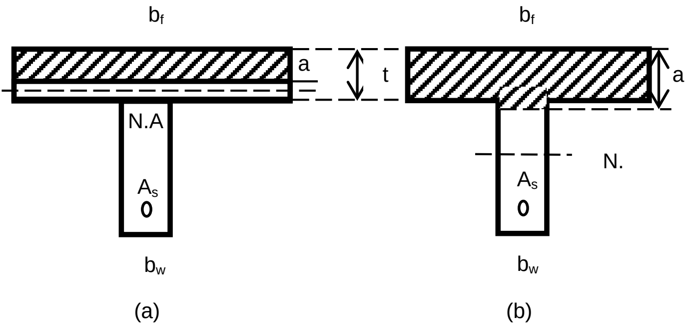
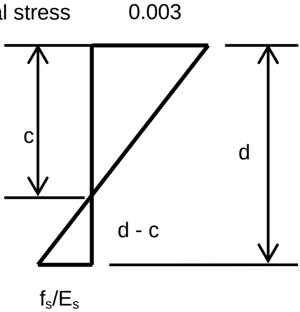


Figure 2 – 3: Location of neutral axis

Tension Steel Stress

In analysis of beams (whatever shape it is), once the value of c is known, the actual stress in tension steel can be computed using Eq. 2 – 29.



From the diagram shown

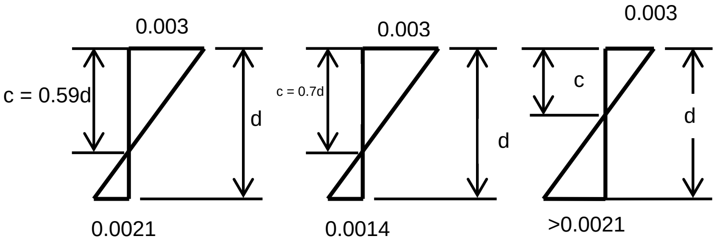
$$\frac{f_s/E_s}{d - c} = \frac{0.003}{c} = E_s = 200,000$$

$$\frac{f_s}{200,000} = \frac{0.003\,(d - c)}{c}$$

$$f_s = 600\ \frac{d - c}{c} \qquad \text{Eq. 2 – 29}$$

The c/d Ratio

One can actually predict when steel will yield once the value of c is known. Note that the strain in concrete is taken as 0.003 and the strain in steel is f_s/E_s. For f_y = 415 MPa, the maximum strain ε_s = 415/200, 000 = 0.0021, and for f_y = 276 MPa, ε_s = 0.0014.



(a) c/d = 0.5 (b) c/d = 0.7 (c)

Figure 2 – 4: Location of neutral axis

As shown in Figure (a), the grade 415 steel will not yield if c/d is greater than 0.59 and will yield if c/d is less than 0.59. The grade 276 steel as shown in Figure (b) will yield if c/d is less than 0.7. Since the maximum steel strength commonly used in construction is the grade 415 (f_y = 415 MPa), we can therefore conclude that if c/d is less than 0.59, the tension steel will yield.

In T – beams where the flange is in compression, the c/d ratio is usually that shown in Figure (c), which easily lead us to a conclusion that he steel yields.

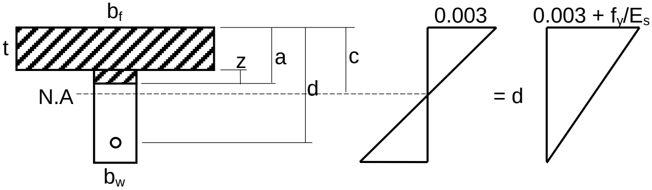
Balanced and Maximum Steel Area

If a is less that the slab thickness t, the balanced steel ration is computed using the balanced ρ in Page 98. If a is greater than t, the following formula will be used.

From the strain diagram shown in the Figure below

$$\frac{c}{0.003} = \frac{d-c}{f_y/E_s} \; ; E_s = 200000 \text{ MPa}$$

$$c = \frac{600\, d}{600 + f_y} \; ; a = \beta_1\, c = \beta_1 = \frac{600\, d}{600 + f_y}$$



Strain diagram in balanced condition

$[T = C]$

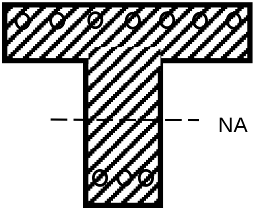
$$A_{sb}\, f_y = 0.85\, f_c\ [b_1 t + b_w z]$$

$$A_{sb} = \frac{0.85\, f'_c\ [b_1\, t + (a\text{-}t) b_w]}{f_y} \qquad \text{Eq.2-30}$$

$$A_{s\ max} = 0.75\ A_{sb} \qquad \text{Eq.2-31}$$

Note Eq. 2 – 30 apply only if a > t.

Design of T-Beams with Negative Moments



When T-beams are resisting negative moments so that their flanges are in tension and the bottoms of their stems are in compression, the formulas for rectangular beams will be applied. The following code requirements shall be applied for this case:

5.10.6.6 Where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in Sec. 5.8.10 or a width equal to 1/10 the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

The intention of this section is to minimize the possibilities of flexural cracks that will occur at the top face of the flange due to negative moments.

<u>Minimum Steel Ratio for T-Beams</u>

Section 5.10.5.1 of NSCP provides that the minimum steel ratio be $1.4/f_y$. It also states that in T-beams where the web is in tension, the ratio ρ shall be computed for this purpose using width of web.

In checking for maximum ρ (ρ_{max}), use $\rho = \frac{A_s}{b_f\, d}$

(only if $a < t$)

In checking for minimum ρ (ρ_{min}), use $\rho = \frac{A_s}{b_w\, d}$

Code Requirements for T-beams (Section 5.8.10)

1. In T-beam construction, the flange and web shall be built integrally or otherwise effectively bonded together.
2. The width of the slab effective as a T-beam shall not exceed ¼ of the span of the beam,

and the effective overhanging flange on each side of the web shall not exceed:

(a) 8 times the slab thickness, and

(b) ½ the clear distance to the next web.

3. For beams with slab on one side only, the effective overhanging flange shall not exceed:

(a) 1/12 the span length of the beam,

(b) 6 times the slab thickness, and

(c) ½ the clear distance to the next web.

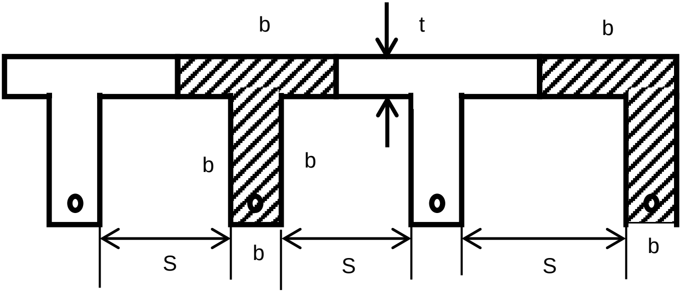


Figure 2 – 5: Effective flange width

For Interior Beam

b_f is the smallest of:

1. $b_f = L/4$
2. $b_f = 16t + b_w$
3. $b_f = S_1/2 + S_2/2 + b_w$

For End Beam

b'_f is the smallest of:

1. $b'_f = L/12 + b'_w$
2. $b'_f = 6t + b'_w$
3. $b'_f = S_3/2 + b'_w$

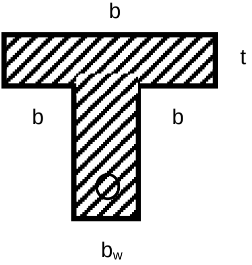
For symmetrical interior beam ($S_1 = S_2 = S$)

b_f is the smallest of:

1. $b_f = L/4$
2. $b_f = 16t + b_w$
3. b_f = center-to-center spacing of beams

4. Isolated beams in which T-shape are used to provide a flange for additional compression area shall have a flange thickness not less than ½ the width of the web and an effective flange width not more than four times the width of the web.

$t \geq b_w/2$
$b \leq 4\ b_w$



5. Where primary flexural reinforcement in a

slab that is considered as a T-beam flange is parallel to the beam, reinforcement perpendicular to the beam shall be provided in the top of the slab in accordance with the following:

(a) Transverse reinforcement shall be designed to carry the factored load on the overhanging slab width assumed to act as a cantilever. For isolated beam, the full width of the overhanging flange shall be considered. For other T-beams, only the effective overhanging slab needs to be considered.

(b) Transverse reinforcement shall be spaced not farther apart than five times the slab thickness, or 450 mm.

STEPS IN DETERMINING THE TENSION STEEL AREA A_s OF T – BEAM WITH KNOWN M_u AND OTHER BEAM PROPERTIES:

I. Assume that the entire flange is in compression and solve for M_{u1}:

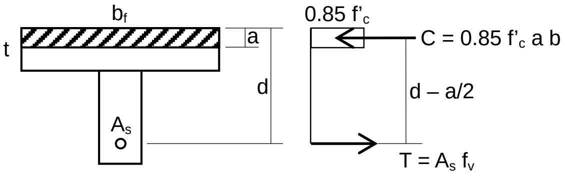
$M_{u1} = \Phi\ C\ (d - t/2)$

$M_{u1} = \Phi\ 0.85\ f_c\ b_f\ t\ (d - t/2) =$

If $M_{u1} > M_{u'}$ then $a < t$, proceed to Step II

If $M_{u1} < M_{u'}$ then $a > t$, proceed to Step III

II. $a < t$



Solve for a:

$M_u = \Phi\, C\,(d - a/2)$
$M_u = \Phi\, 0.85\, f_c\, ab\,(d - a/2)$; a =

$[T = C] = A_s f_y = 0.85\, f_c\, ab$; A_s =

Solve for ρ_{max} and compare with $A_s/\, b_f\, d$

If $A_s/\, b_f\, d < \rho_{max}$, design is OK

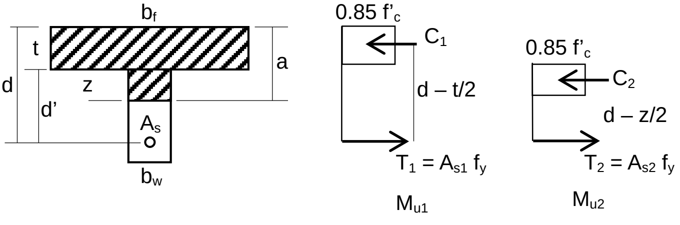
If $A_s/\, b_f\, d > \rho_{max}$, the beam needs compression steel (this seldom happen)

Solve for $\rho_{max} = 1.4 / f_y$ and compare with $A_s/\, b_w\, d$

If $A_s/\, b_w\, d < \rho_{min}$, design is OK

If $A_s/\, b_w\, d > \rho_{min}$, use $\rho = \rho_{min}$ (this seldom happen)

Use $A_s = \rho_{min}\, b_w\, d$



$M_u = M_{u1} + M_{u2}$

Where M_{u1} = the same value in Step 1

$M_{u2} = M_u - M_{u1} =$

$$M_{u2} = \Phi\ C\ (d - z/2)$$
$$M_{u2} = \Phi\ 0.85\ f_c\ b_w\ z\ (d' - z/2)$$
$$z =$$

[T = C]

$$A_s\ f_y = C_1 + C_2$$
$$A_s\ f_y = 0.85\ f_c\ b\ t + 0.85\ f_c\ b_w\ z$$
$$A_s =$$

Solve for $\rho_{max} = 1.4 / f_y$ and compare with $A_s / b_w\ d$

If $A_s / b_w\ d > \rho_{min}$, design is OK

If $A_s / b_w\ d < \rho_{min}$, use $\rho = \rho_{min}$ (this seldom happen)

Use $A_s = \rho_{min}\ b_w\ d$

Solve for $A_{s\ max}$:

$$a = \beta_1 (600d / f_y + 600)$$

$$A_{s\ max} = 0.75\ A_{sb} = 0.75\ \frac{0.85\ f'_c\ [b_1 t + (a - t) b_w]}{f_y}$$

If $A_s < A_{s\ max}$, value is OK
If $A_s > A_{s\ max}$, the beam needs compression steel (this seldom happens to T-beam)

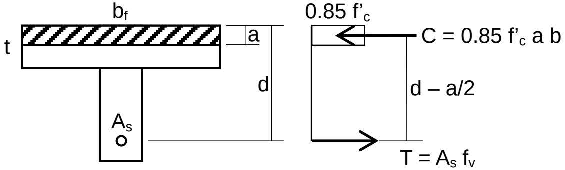
STEPS IN DETERMINING M_U OF A T-BEAM WITH GIVEN A_S AND OTHER BEAM PROPERTIES:

I. With tension steel yielding ($f_s = f_y$), compute the area of compression concrete, A_c.

[C = T] $0.85\ f'_c\ A_c = A_s\ f_y$; $A_c =$

Area of compression flange, $A_f = b_f\ t$
If $A_c < A_f$, $a < t$, proceed to Step II
If $A_c > A_f$, $a > t$, proceed to Step III

II. $a < t$



Solve for a:

$A_c = b_f \times a$; $a =$

$M_u = \Phi\ T\ (d - a/2)$
$M_u = \Phi\ A_s\ f_y\ (d - a/2)$

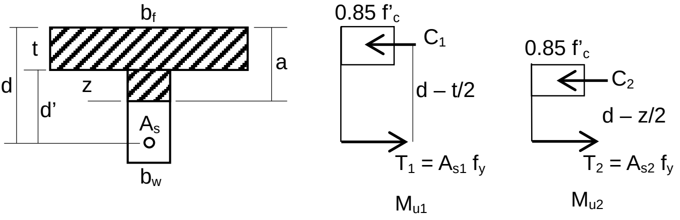
Verify if steel yields (this may not be necessary anymore)

$c = a/\beta_1$

$f_s = \dfrac{600(d - c)}{c} =$

If $f_s > f_y$, steel yields (assumption is correct)
If $f_s < f_y$, steel does not yield (this seldom happen)

III. $a > t$



$M_u = M_{u1} + M_{u2}$

Solve for z:

$A_c = A_f + b_w z$ (See Step I for the values of A_c and A_f
$z =$

Verify if steel yields

$a = t + z =$

$c = a/\beta_1 = \quad f_s = \frac{600(d - c)}{c} =$

If $f_s > f_y$, steel yields (assumption is correct)
If $f_s < f_y$, steel does not yield (this seldom happen)

$M_{u1} = \Phi\, C_1\, (d – t/2) = \Phi\, 0.85\, f_c\;\; A_f\, (d – t/2)$
$M_{u2} = \Phi\, C_2\, (d' – z/2) = \Phi\, 0.85\, b_w\;\; z\, (d' – z/2)$
$M_u = M_{u1} + M_{u2} =$

BEAM DEFLECTION (SECTION 5.9.5)

Sect. 5.9.5.2.2 Where deflections are to be computed, deflections that occur immediately on application of load shall be computed by usual methods or formulas for elastic deflections, considering effects of cracking and reinforcement on member stiffness

Sect. 5.9.5.2.3 Unless stiffness values are obtained by a more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity E_c for concrete and with the effective moment of inertia as follows, but not greater than I_g.

$$\boxed{I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \qquad \text{Eq. 2 - 32}}$$

Where

$M_{cr} = \frac{f_r\, I_g}{y_t}$

f_r = modulus of rapture of concrete, MPa, for normal weight concrete, $f_r = 0.7\sqrt{f'_c}$

M_a = maximum moment in member at stage deflection is computed.

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.

I_{cr} = moment of inertia of cracked section transformed to concrete

Y_t = distance from centroidal axis to gross section, neglecting reinforcement, to extreme fiber in tension.

When lightweight aggregate is used, one of the following modifications shall apply:

(a) When f_{ct} is specified and concrete is proportioned in accordance with Sec. 5.5.2, f_r shall be modified by substituting 1.8 f_{ct} for $\sqrt{f'_c}$ but the value of 1.8 f_{ct} shall not exceed $\sqrt{f'_c}$.

(b) When f_{ct} is not specified, f_r shall be multiplied by 0.75 for all lightweight concrete, and 0.85 for sand-lightweight concrete. Linear interpolation is permitted if partial sand replacement is used.

Sect. 5.9.5.2.4 For continuous members, effective moment of inertia may be taken as average of values obtained from Eq. 2 – 32 for the critical positive and negative moment sections. For prismatic members, effective moment of inertia may be taken as the value obtained from the Eq. 2 – 32 at midspan for simple and continuous spans, and at the support for cantilevers.

Sect. 5.9.5.2.5 Unless values are obtained by a more comprehensive analysis, additional long-term deflection resulting from creep and shrinkage of flexural members (normal weight or lightweight concrete) shall be determined by multiplying the immediate deflection caused by the sustained load considered, by the factor

$$\lambda = \frac{\xi}{1+50\rho'} \qquad \text{Eq. 2 - 33}$$

Where ρ' shall be the value of reinforcement ratio for non-prestressed compression reinforcement at midspan for simple and continuous spans, and at support for cantilevers. It is permitted to assume the time-dependent factor ξ for sustained loads to be equal to

5 years or more 2.0
12 months 1.4
6 months 1.2
3 months 1.0

5.9.5.2.6 Deflection computed in accordance with Sec. 5.9.5.2.2 through Sec. 5.9.5.2.5 shall not exceed limits stipulated in Table 2 – 5

Table 2 – 5: Maximum Permissible Computed Deflections

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load LL	L/180
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load LL	L/360
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-time deflection due to all sustained loads and the immediate deflection due to	L/480 **
Floors not supporting or		L/240 ****

attached to nonstructural elements likely to be damaged by large deflections	any additional live load)***	

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflections, including added deflections due to ponded water and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

** Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

*** Long-time deflection shall be determined in accordance with Sec. 5.9.5.2.5 or Sec. 5.9.5.4.2 but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

**** But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

NSCP COEFFICIENTS FOR CONTINUOUS BEAMS AND SLABS

Section 5.8.3.3 of NSCP states that in lieu of frame analysis, the following approximate moment and shears are permitted for design of continuous beams and one-way slabs (slabs reinforced to resist flexural stresses in only one direction) provided:

(a) There are two or more spans,

(b) Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent,

(c) Loads are uniformly distributed,

(d) Unit live loads does not exceed three times unit dead load and

(e) Members are prismatic

Positive moment

End spans

Discontinuous end unrestrained …………………..… $W_u L_n^2/11$

Discontinuous end integral with support …….…….. $W_u L_n^2/14$

Interior spans ……………………………………........ $W_u L_n^2/16$

Negative moments at exterior face of first interior support

Two spans ……………………………………………….. $W_u L_n^2/9$

More than two spans ………………………………..…. $W_u L_n^2/10$

Negative moments at other faces of interior

Supports ………………………………………………………... W_u $L_n^2/11$

Negative moment at other face of all supports for:
Slabs with spans not exceeding 3 m; and beams
Where ratio of sum of column stiffness to beam
Stiffness exceeds eight at each end of the span ………... W_u $L_n^2/12$

Negative moment at interior face of exterior
Supports for members built integrally with supports
Where support is a spandrel beam ………………………. W_u $L_n^2/24$
When support is a beam ……………………..…………….. W_u $L_n^2/16$

Shear in end members at face of first interior su……………....1.15 w_u $L_n/2$
Shear at face of all other supports….. W_u $L_n/2$
Where L_n = clear span for positive moment or shear and average of adjacent clear spans for negative moment.

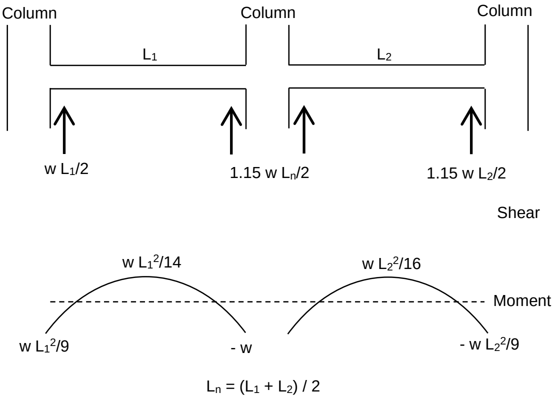


Figure 2 – 6: Shear and moment for continuous beam or slab with spans ad discontinuous end integral with support

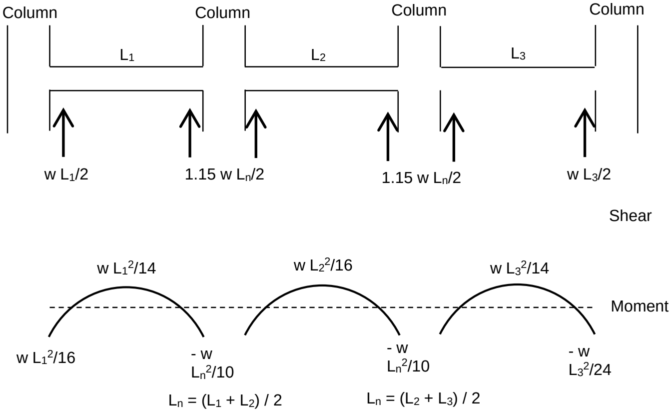


Figure 2 – 7: Shear and moment for continuous beam or slab with more than two discontinuous end integral with support

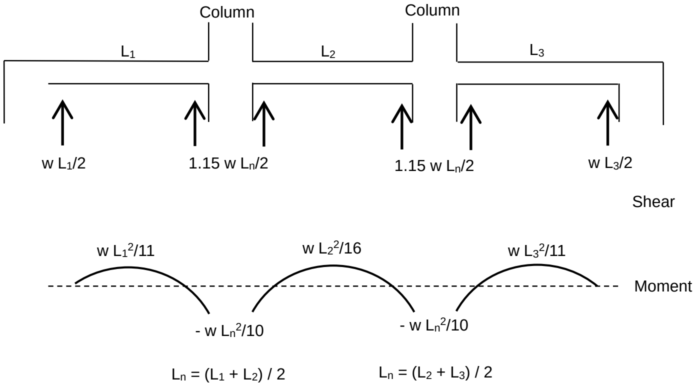


Figure 2 – 8: Shear and moment for continuous beam or slab with more than two discontinuous and unrestrained

<u>ONE-WAY SLAB</u>

Reinforced concrete slab are large flat plates that are supported at its sides by reinforced concrete beams, walls, columns, steel beams, or by the ground. If a slab is supported on two opposite sides only, they are referred to one-way slabs since the bending occurs in one direction only. If the slab is supported on all four sides, it is called two-way slab since the bending occurs in both direction.

If a rectangular slab is supported in all four sides but the long side is two or more times the short side, the slab will, for all practical purposes, act as one way slab, with bending occurring in the short direction.

A one-way slab is considered as a wide, shallow, rectangular beam. The reinforcing steel is usually spaced uniformly over its width. One-way slabs are analyzed by considering a one-meter strip, which is assumed independent of the adjacent strips. This method of analysis is somewhat conservative because we neglected the lateral restraint provided by the adjacent strips.

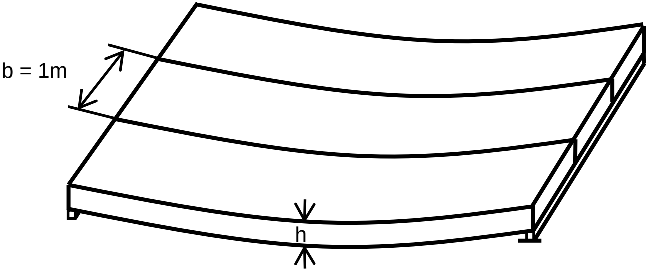


Figure 2 – 9: One-way slabs on simple support

Minimum Spacing of Reinforcement

According to Section 5.7.6.5, the flexural reinforcement shall not be spaced farther apart than 3 times the slab thickness, nor 450 mm.

Shrinkage and Temperature Reinforcement, ρ_t

Concrete shrinks as it hardens. In addition, temperature changes occur that causes expansion and contraction of concrete. In this effect, the code (5.7.12) requires that one-way slabs, where flexural reinforcement extends in one direction only, should be reinforced for shrinkage and temperature stresses perpendicular to flexural reinforcement. According to Section 5.7.12.2.1, the area of shrinkage reinforcement shall provide at least the following ratios of gross concrete area bh, (where h is the slab thickness) but not less than 0.0014.

(a) Where Grade 275 deformed bars are used 0.0020

(b) Where Grade 415 deformed bars or welded wire fabric (plain or deformed) are used ………... 0.0018

(c) Where reinforcement with $f_y > 415$ MPa measured at yield strain of 0.35% are used ……………………….. $\frac{0.0018 \times 400}{f_y}$

Shrinkage and temperature reinforcement may not be spaced not farther apart than 5 times the slab thickness, or 450 mm (Section 5.7.12.2.2).

Steps in the Design of One-Way Slabs

I. Identify the uniform floor pressure (Pa) to be carried by the slab. These loads consist of:

1.) Live load pressure, LL (Pa)
2.) Dead load pressure, DL (Pa)

3.) Ceiling load (below the slab), DL (Pa)

II. Determine the minimum slab thickness h from Table 2 – 4. If necessary adjust this value depending on your judgment.

III. Compute the weight of the slab; weight = γ_{conc} x h, DL (Pa)

IV. Calculate the factored moment (M_u) to be carried by the slab.

Factored floor pressure = 1.4DL + 1.7LL
Uniform load, W_u = factored pressure x 1 m

V. Compute the effective depth, d:

d = h – covering (usually 20 mm) – ½ (main bar diameter)

VI. Compute the required ρ:

Solve for R_u: $M_u = \Phi R_u b d^2$ where b = 1000 mm

$$\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2 R_u}{0.85 f'_c}}\right]$$

Solve for ρ_{max} and ρ_{min}

If ρ is less than ρ_{max} and ρ_{min}, use ρ
If ρ is greater than ρ_{max}, increase the depth of slab to ensure ductile failure

If ρ is less than ρ_{min} use $\rho = \rho_{min}$

VII. Compute the required main bar spacing

$$A_s = \rho\, b\, d = \rho\,(1000)\, d$$

$$S_1 = \frac{A_{bar}}{A_s} \times 1000$$

Use the smallest of the following for the main bar spacing:

(a) S_1, (b) 3 x h, and (c) 450 mm

VIII. Temperature bars: See page 129 for the spacing:

$$A_{st} = \rho_t\, b\, h$$

$$S_2 = \frac{A_{bar}}{A_s} \times 1000$$

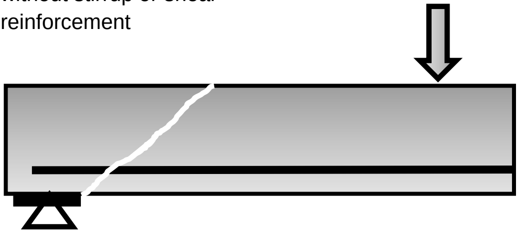
Use the smallest of the following for temperature bar spacing:

(a) S_2, (b) 5 x h, and (c) 450 mm

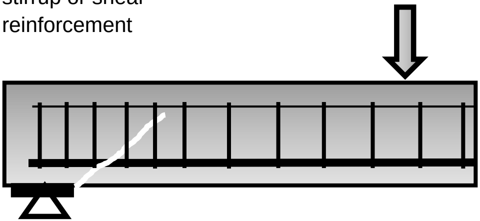
SHEAR AND DIAGONAL TENSION

Another type of beam failure other than bending is shear failure. Shear failures are very dangerous especially if it happens before flexure failure because they can occur without warning. To avoid shear failure, the Code provides permissible shear values that have larger safety factors compared to bending failure, thus ensuring ductile type of failure.

(a) Shear failure of beam without stirrup or shear reinforcement



(b) Shear failure of beam stirrup or shear reinforcement



Without stirrup, there is nothing to stop the concrete from splitting due to diagonal tension as in Figure (a). Stirrups prevent this occurrence especially if they are closely spaced as in Figure (b).

Basic Code Requirements

The basic Code requirement (Sec. 5.11.1) on shear strength is that the factored shear forced V_u shall be equal or less than the design shear $\text{Ø} V_n$, or

$$V_u \leq \text{Ø} V_n \qquad \text{Eq. 2 – 34}$$
$$V_n = V_c + V_s \qquad \text{Eq. 2 – 35}$$

For a beam with no web reinforcement, the shearing force that causes the first diagonal cracking can be taken as the shear capacity of the beam. For a beam that does contain constant amount of shear force V_c, and the web reinforcement need only be designed for the shear force V_s in excess of that carried by the concrete, or

$$V_s = V_n - V_c \qquad \text{Eq. 2 – 36}$$

The amount of shear V_c that can be carried by concrete at ultimate is at least equal to the amount of the shear that would cause diagonal cracking. The amount of shear provided by the reinforcement V_s is calculated using the truss analogy with a 45° inclination of the diagonal members.

Shear Strength Provided by Concrete, V_c

According to Section 5.11.3.1 the shear strength provided by concrete subject to shear and flexure only is:

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w\, d \qquad \text{Eq. 2 – 37}$$

Or in more detailed calculation (Section 5.11.3.2.1)

$$V_c = \left[\left(\sqrt{f'_c} + 120\,\rho_w\,\frac{V_u\,d}{M_u}\right) \div 7\right] b_w\,d \le 0.3\,\sqrt{f'_c}\,b_w\,d \qquad \text{Eq. 2 – 38}$$

Where $\sqrt{f'_c}$ is in MPa and shall not exceed 0.7 MPa except as provided by Section 5.11.1.2.1, b_w is the width of web in mm, d is the effective depth in mm, and $\rho_w = A_s/b_w\,d$. The quantity $V_u\,d/M_u$ in Eq. 2 – 38 shall not be taken greater than 1.

<u>Types of Shear Reinforcement</u>

According to Section 5.11.5.1 of the Code, shear reinforcement may consist of:

a) Stirrups perpendicular to axis of member, and
b) Welded wire fabric with wires located perpendicular to axis of member.

For nonprestressed members, shear reinforcement may also consist of:

a) Stirrups making an angle of 45° or more with longitudinal tension reinforcement,
b) Longitudinal reinforcement with bent portion making an angle of 30° or more with the longitudinal tension reinforcement,
c) Combinations of stirrups and bent longitudinal reinforcement, and
d) Spirals.

Design Yield Strength of Stirrups

According the Section 5.11.5.2 the design yield strength of shear reinforcement shall not exceed 415 MPa. Stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends to develop the design yield strength of reinforcement.

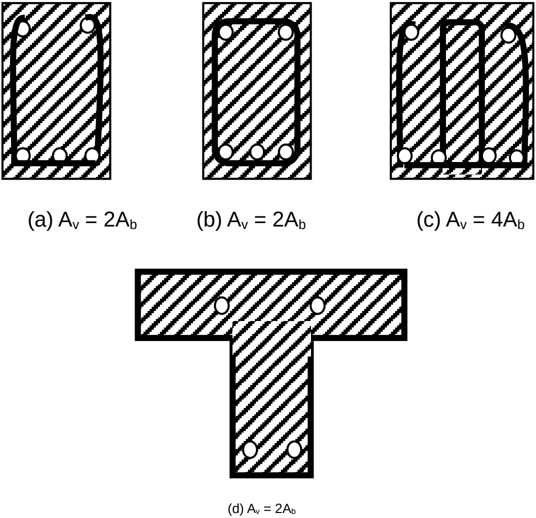


Figure 2 – 11: Types of stirrups

Spacing Limits of Shear Reinforcement, s

According to Section 5.11.5.4 of the Code, the spacing s of shear reinforcement placed perpendicular to axis of members shall not exceed d/2 in nonprestressed members and (3/4)h in prestressed members, nor 600 mm. Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45° line, extending toward the reaction from mid-depth of members d/2 to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement.

When V_s exceed $\frac{1}{3}\sqrt{f'_c}\, b_w d$ maximum spacing given by the above limits shall be reduced by one-half.

Minimum Shear Reinforcement

According to Section 5.11.5.5 of the Code, a minimum area of shear reinforcement shall be provided in all reinforced concrete flexural members (prestressed and nonprestressed) where factored shear force V_u exceeds one-half the shear strength provided by concrete ϕV_c, except:

- (a) Slabs and footings
- (b) Concrete joist construction defined by Sec. 5.8.11
- (c) Beams with total depth not greater than 250 mm, $2^1/_2$ times thickness of flange, or 1/2 the width of web, whichever is greatest.

This minimum shear reinforcement requirement may not be required if shown by test that required nominal flexural and shear strength can be developed when shear reinforcement is omitted. Such tests shall simulate effect of different settlement, creep, shrinkage, and temperature change, based on a realistic assessment of such effects occurring in service.

Where shear reinforcement is required, the minimum area of shear reinforcement shall be computed by

$$A_v = \frac{b_w\, s}{3\, f_y} \qquad \text{Eq. 2 - 39}$$

Where b_w and s are in millimeters.

Shear Strength Provided by Reinforcement

When factored shear V_u exceeds strength ϕV_c, shear reinforcement shall be provided to satisfy Eq. 2-34 and Eq. 2-35. The shear strength provided by the stirrups is given by the following but shall not be taken greater $\frac{1}{3}\sqrt{f'_c}\, b_w\, d$.

(a) When shear reinforcement perpendicular to axis of member is used.

$$V_s = \frac{A_v\, f_y\, d}{s} \qquad \text{Eq. 2 - 40}$$

Where A_v is the area of shear reinforcement within a distance s.

(b) When inclined stirrups are used as shear reinforcement.

$$V_s = \frac{A_v\, f_y\, (\sin\alpha + \cos\alpha)\, d}{s} \qquad \text{Eq. 2 - 41}$$

Where α is the angle between inclined stirrups and longitudinal axis of member.

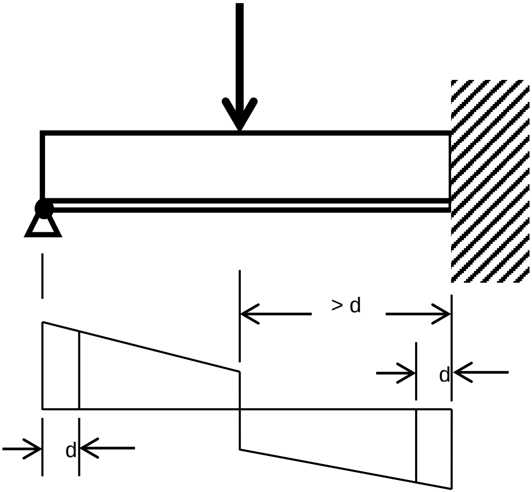
(c) When shear reinforcement consist of a single bar or a single group of parallel bars, all bent up at the same distance from the support,

$$V_s = A_v f_y \sin\alpha \leq \frac{1}{4}\sqrt{f'_c}\, b_w d \qquad \text{Eq. 2 - 42}$$

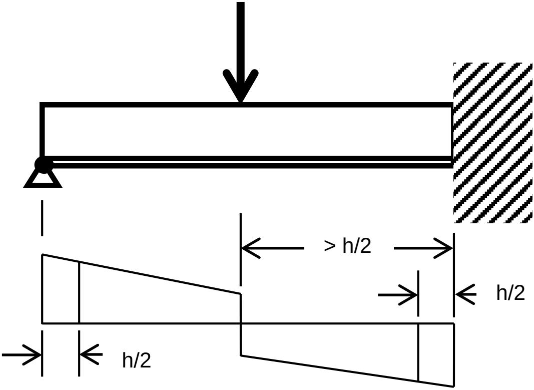
Critical Section for Beam Shear

According to section 5.11.1.3 of NSCP, the maximum factored shear force V_u at supports may be computed in accordance with the following conditions provided that:
(a) The support reaction, in direction of the applied shear, introduces compression into the end regions of member, and
(b) No concentrated load occurs between the face of the support and the location of the critical section.

1. For non-prestressed members, sections located less than a distance d from face of support may be designed for the same shear V_u as that computed at a distance, d.



2. For prestressed member, sections located less than a distance h/2 from face of support may be designed for the same shear V_u as that computed at a distance h/2.



Steps in Vertical Stirrup Design

I. Calculate the factored shear force V_u at critical sections defined in Page 136, or at any section you want the spacing to be determined.

II. Calculate the shear strength provided by concrete, V_c.

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w\, d \qquad \text{(or using Eq. 2 – 38)}$$

If $V_u > \Phi V_c$, stirrups are necessary, proceed to step III.
If $V_u < \Phi V_c$, but $V_u > ½ \phi V_c$, proceed to step V
(Sec. 5.11.5.5.1)
If $V_u > ½ \Phi V_c$, stirrups are not needed

III. Calculate the shear strength V_s to be provided by the stirrup

1. $V_n = V_u/\Phi$
2. $V_s = V_n - V_c = V_u/\Phi - V_c$

If $V_s \leq \frac{2}{3}\sqrt{f'_c}\, b_w\, d$, proceed to Step IV (Sect. 5.11.5.6.8)

If $V_s > \frac{2}{3}\sqrt{f'_c}\, b_w\, d$, adjust the size of the beam (Sect. 5.11.5.6.8)

IV. Spacing of stirrups:

Spacing, $s = \frac{A_v\, f_y\, d}{V_s}$; See Figure 2 – 11 in Page 134 for the value of A_v.

If s < 25 mm, increase the value of A_v by either using a bigger bar size or adding more shear area.

Maximum spacing, s:

(a) When $V_s \leq \frac{1}{3}\sqrt{f'_c}\, b_w\, d$, S_{max} = d/2 or 600 mm

(b) When $V_s > \frac{1}{3}\sqrt{f'_c}\, b_w\, d$, S_{max} = d/4 or 300 mm

V. If $V_u < \phi\, V_c$ but $V_u > ½\, \phi\, V_c$

Minimum area of stirrup. $A_v = \frac{b_w\, s}{3\, f_y}$ (Sect. 5.11.5.5.3)

Where s = d/2 or 600 mm (whichever is smaller)

BOND, DEVELOPMENT LENGTH, HOOKS, AND SPLICING FOR REINFORCEMENT

Bond

In reinforced concrete we assumed that the concrete and steel work as a unit. For this to happen there must be absolutely no slippage of the bars in relation to the surrounding concrete. The steel and concrete must stick or bend together for them to act as a unit. If there is slipping of steel with respect to surrounding concrete, there will be no transfer of stress from steel to concrete and vice versa and as a result, the concrete will act as an unreinforced member and will be subject to collapse.

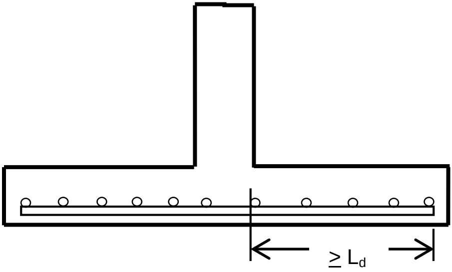


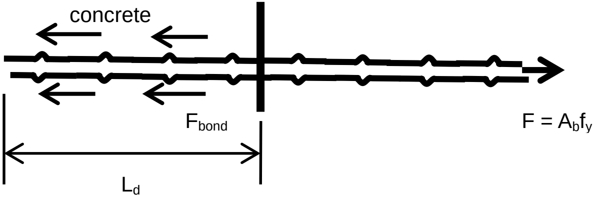
Figure 2 -12 Development of bars in footing

Development Length of Straight bars

Bar development length L_d is the embedment necessary to assure that the bar can be stressed to its yield point with some reserved to insure member toughness. Development length is a function of bar diameter d_b, yield point f_y, and concrete strength f'_c. Other items affecting the development length are bar spacing, concrete cover and transverse reinforcement.

Basic Concept of Development Length

In the basic concept of anchorage length, a bar is embedded in a mass of concrete as shown. Under initial loading, the actual bond stress will be larger near the surface and nearly zero at the embedded end. Near failure, the bond stress along the bar will be more uniformly distributed. If the average bond stress at ultimate is u, then



$F = F_{bond}$

$A_v\, f_y = u \times \pi\, d_b\, L_d$

$\frac{\pi}{4}\, d_b^2\, f_y = u \times \pi\, d_b\, L_d$

$$\boxed{L_d = \frac{f_y}{4\,u} d_b \qquad\qquad \text{Eq. } 2-43}$$

According to Section 5.12.1, calculated tension or compression in reinforcement at each section of reinforced concrete members shall be developed on each side of that section by embedment length, hook or mechanical device, or a combination thereof. Hooks may be used in developing bars in tension only.

The Code provides the basic development length l_{db} for various situations. The values provided by the code have to be modified for different condition. Thus, the minimum development length L_d required by the code can be expressed as

$$L_d = l_{db} \times \text{applicable modification factor(s), m} \qquad \text{Eq. 2 – 44}$$

but shall not be less than 300 mm, except for the lengths required for tension lap splices and for the development of shear reinforcing.

Basic Development Length of Bars in Tension

According to Section 5.12.2 of the Code, the basic development shall be:

For 32 mm bar & smaller and deformed wire:

$$l_{db} = \frac{0.02\, A_b\, f_y}{\sqrt{f'_c}} \geq 0.06\, d_b\, f_y \qquad \text{Eq. 2 – 45}$$

For 36 mm bar:

$$l_{db} = 25\, f_y / \sqrt{f'_c} \qquad \text{Eq. 2 – 46}$$

For deformed wire:

$$l_{db} = 3\, d_b f_y / 8 \sqrt{f'_c} \qquad \text{Eq. 2 – 47}$$

Modification Factors for Bars in Tension

Basic development length l_{db} shall be multiplied by the applicable modification factors m for the following conditions:

Condition	
(a) For bars in beams or columns with a minimum cover not less than specified the Code:	1.0
(b) Bars in beams or columns with transverse reinforcement satisfying tie requirements of Code:	1.0
(c) Bars n beams of columns with clear spacing of not less than $3d_b$:	1.0
(d) Bars in the inner layer of slabs or wall reinforcement and with clear spacing of not less than $3d_b$:	1.0
(e) Any bars with cover of not less than $2d_b$ and with clear spacing of not less than $3d_b$:	1.0
(f) For bars with cover d_b or less or with clear spacing of $2d_b$ or less:	2.0
(g) For bars not included in items a to f:	1.4
(h) For 32 mm bars and smaller with clear spacing not less than $5d_b$ and with cover from face of member to edge bar, measured in the plane of the bars, not less than $2.5d_b$, the factors in items	

a to g may be multiplied by 0.8	
(i) Top reinforcement:	1.3
(j) Lightweight aggregate concrete:	1.3
(k) Lightweight aggregate when f_{ct} is specified:	$\frac{\sqrt{f'_c}}{(1.8\ f_{ct})}$
(l) For reinforcement enclosed within special reinforcement not less than 6 mm diameter and not more than 100 mm pitch, within 12 mm or larger circular ties spaced at not more than 100 mm on center or within 12 mm larger ties or stirrups spaced not more than 100 mm on center and arranged such that alternate bars such have support provided by the corner of a tie hoop with an included angle of not more than 135° the factors in items a through g may be multiplied by **1.8**.	
(m) **Excess Reinforcement.** Development length may be reduced where reinforcement in a flexural member is more than required by analysis by a factor.	$\frac{A_s \text{ required}}{A_s \text{ provided}}$

Basic Development of Bars in Compression

According to Section 5.12.3.2 of the Code, the basic development for bars in compression shall be:

$$l_{db} = \frac{0.24\ d_b f_y}{\sqrt{f'_c}} \geq 0.04\ d_b f_y \qquad \text{Eq. 2 – 48}$$

Modification Factors for Bars in Compression

Basic development length l_{db} may be multiplied by the applicable factors for:

Condition	Modification Factor, m
(a) **Excess Reinforcement.** Reinforcement more than required by the analysis	$\dfrac{A_s \text{ required}}{A_s \text{ provided}}$
(b) Spirals and Ties. Reinforcement enclosed within spiral reinforcement not less than 6 mm diameter and not more than 100 mm pitch or within 10 mm ties and spaced at not more than 100 mm on center	0.75

Development of Bundled Bars

Development length of individual bars within a bundled, in tension or compression, shall be that for the individual bar, increased 20 percent for three-bar bundle, and 33 percent for four-bar bundle.

For determining the appropriate modification factors, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

Development of Flexural Reinforcement (Sec. 5.12.10)

Tension reinforcement in flexural members may be developed by:

(a) Bending across the web to be anchored or
(b) Made continues with reinforcement on the opposite face of member.

Critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent. Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of member or $12d_{b,}$ whichever is greater, except at supports of simple spans and at free end of cantilever. Continuing reinforcement shall have an embedment length not less than the development length l_d beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

Flexural reinforcement shall not be terminated in a tension zone unless one of the following conditions is satisfied:

a.) Shear at the cutoff point does not exceed two-thirds that permitted including shear strength of shear reinforcement provided.

b.) Stirrup area in excess of that required fir shear and torsion is provided along each terminated bar or wire over a distance from the termination point equal to the three-fourths the effective depth of member. Excess stirrup area A_v shall not be less than 0.4 b_w s/f_y. Spacing s shall not exceed d/8 β_b where β_b is the ratio of the area of the reinforcement at the section.

c.) For 32-mm bar and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and shear does not exceed three-fourths that permitted.

Adequate anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as sloped, stepped, or tapered footings; brackets; deep flexural members; or members in which tension reinforcement is not parallel to compression face.

Development of Positive Moment bars

According to Section 5.12.11 of the Code, at least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of member into the support. In beams, such reinforcement shall extend into the support at least 150 mm.

At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d computed by Eq. 2 – 44 need not exceed Eq. 2 – 49.the purpose of this limitation is to keep bond stresses within reason at these points of low moments and large shears.

$$L_d \leq \frac{M_n}{V_u} + l_a \qquad \text{Eq. 2 – 49}$$

Where:

M_n is nominal moment strength assuming all reinforcement at the section to be stressed to the specified yield strength f_y

V_u is factored shear force at the section (at point of support for simple support and at point of inflection for continuous beam)

l_a at a support shall be embedment length beyond center of support

l_a at a point of inflection shall be limited to the effective depth of member or $12d_b$, whichever is greater.

Value of M_n/V_u may be increased 30 percent when the ends of reinforcement are confined by a compressive reaction such as where there is a column below but not when a beam frames into a girder i.e.

$$L_d \leq 1.3 \frac{M_n}{V_u} + l_a \qquad \text{Eq. 2 – 50}$$

When L_d computed by Eq. 2 – 44 exceed Eq. 2 – 49 or Eq. 2 – 50, use a smaller bar size, or increase the value of the end anchorage l_a as by the use of hooks.

Development of Negative Moment Reinforcement

Negative – moment reinforcement should have an embedment length into the span to develop the calculated tension in the bar, or a length equal to the effective depth of the member, or $12d_b$ whichever is greatest. At least one – third of the total negative reinforcement should have an embedment length beyond the point of inflection not less than the effective depth of the member, or $12d_b$, or 1/16 of the clear span whichever is greatest.

Hooks

If sufficient space is not available to anchor tension bars by running them straight for the required development length as required by the Code, hooks may be used.

Development of Standard Hooks

According to Section 5.12.5, the basic development length l_{hb} for standard hooks with f_v = 415 MPa is equal to 100 d_b / f'_c.

The actual development length l_{dh} is taken as the basic development length l_{hb} multiplied by applicable modification factors, but l_{dh} shall not be less than 8 d_b nor less than 150 mm.

Modification Factors (Sect. 5.12.5.3)

1. If the reinforcing bar than an f_y Other than 415 MPa, l_{hb} is to be multiplied by $f_y/415$. (Sec.5.12.5.3)

2. When 90° hooks and 32 mm or smaller bars are used and when 60 mm or more side cover normal to the hook is present, together with at least 50 mm cover for the bar extension, l_{hb} is to be multiplied by 0.70 .(Sec.5.12.5.3.2)

3. When hooks made of 32 mm or smaller bars are enclosed vertically and horizontally within ties or stirrup ties spaced no farther apart than 3db, l_{hb} is to be multiplied by 0.80. (Sec. 5.12.3.3)

4. When the amount of the flexural reinforcement exceed the theoretical amount required and where the specifications being used do not specifically require that developments length be based on f_y the value of l_{hb} multiplied by (As required)/(As provided). (Sec 5.12.5.3.4)

5. When lightweight concrete are used, a modification factor of 1.3 must be applied. (Sec 5.12.5.3.5)

6. For bars being developed by standard hook at discontinuous end of members with both side cover on top (or bottom) cover over hook less than 60 mm hooked bar shall be enclosed within or stirrup ties spaced along the full development length l_{hb} not greater than 3db where d_b is the diameter of hooked bar. For this case, the factor mentioned in 3 shall not apply. (5.12.5.4)

Splices of Reinforcement, General

It is generally necessary to splice bars, partly because of limited length of the commercial bars but more because of the limited length of the commercial bars but more because of the difficulty of interweaving long bars on the job   Splicing may be done by welding. By mechanical connections, or most frequently by lapping bars.   Lapped are usually tied in contact.

Lap Splices

Lap splices shall not be used for bars larger than 32 mm except as provider by the Code. Bars larger than 32 mm lap splices in flexural members shall not be spaced transversely farther apart than 1/5 the required lap splice neither length nor 150 mm.

Welded Splices and Mechanical Connections

Welded splice and other mechanical connections are permitted by the Code. A full welded splices shall have bars butter and welded to develop in tension at least 125 percent of specified yield strength f_y of the bar. A full of mechanical connections shall develop in tension or compression, as required at least 125 percent of specified yield strength f_y of the bar.

Splices in Tension

The minimum length of lap for tension lap splices shall as required for Class A or B splice. But not less than 300 mm where

Class A splice .... 1.0 L_d
Class B splices…….. 1.3 L_d

Where L_d is the tensile development length for the specified yield strength f_y

Lap splices of deformed bars and deformed wire in tension shall be Class B splices except that Class A splices are allowed when

(a) The area of reinforcement provided is at least twice that required by analysis over the entire length of the splice. And

(b) One-half or less of the total reinforcement is spliced within the required lap strength

Welded splices or mechanical connections used where area of reinforcement provided is at least twice that required by the analysis shall meet the following:

(a) splices shall be staggered at least 600 mm and in such manner as to develop at every section but not less than 140 MPa for the total area of reinforcement provided, and

(b) In computing tensile force developed at each portion. Spliced reinforcement maybe rated at that fraction of f_y denied by the ratio of the shorter actual development length to L_d required to develop the specified yield strength $f_{y.}$

<u>Splices of Deformed Bars in Compression</u>

Compression bars may be spliced by lapping, by the end bearing, and by welding or mechanical devices. According to the Section 5.12.16.1. the minimum splice length of such bars should be the development length L_d but may not be less than 0.07fy d_b for f_y of 415 MPa, Should the concrete strength f'_c less than 20 MPa, the length of lap should be increased by one-third.

When bars of different size are lap spliced in compression splice length shall be the larger of development length of larger bar, or splice length of smaller bar.

AXIALLY LOADED COLUMNS

Classification of Columns

In general, columns are classified as short columns and long columns if the height of the column is less than three times

Its least lateral dimension, it may be considered as short compression blocks or pedestal. Pedestals may be designed with reinforcement with a maximum permissible compressive strength of 0.85Ø f_c, where Ø is 0.70 (Sect. 5.10.15), if the compressive strength is greater than this value, the pedestal will have to be designed as a reinforced concrete short column if the reinforced concrete column fails due to the initial material failure. It is classified as short column. The load of the short columns depends on the dimension and the strength of the materials of which it is made if the length of the column is increased. Columns that fail by buckling are called long columns

P-Delta Moment

When a column subjected to primary moment’s m, such as those caused by applied loads or joints rotation, the axis of the member deflects laterally. This deflection additional moment applied to the column, which is equal to the column, load times lateral deflection. This moment called secondary moment or P-Delta moment.

If the secondary moments become too large, the column is said to be long column and it is necessary to design its section from the sum of both primary and secondary moments. However, the

Code permits that columns be design its short columns if the secondary or P∆ effects does not reduce their strength by more than 5%.

(a) Plain concrete pedestal – this may be used only if the height does not exceed three times the least lateral dimension.

(b) Tied columns – A column in which the longitudinal bars are braced with a series of close ties.

(c) Spiral columns – A column in which the longitudinal bars and concrete core are wrapped with a closely spaced helix or spiral.

(d) Composite columns- These columns may contain a structural steel shape surrounded by longitudinal bars with ties or spiral or it may consist of high-strength steel tubing filled with concrete.

Tied and spiral columns are the most common forms. Either type may be circular, octagonal, square, or rectangular section. Tied columns may also be L. T or other irregular shape.

Axial Load Capacity of Columns

Axial load without moment is not practical case in design of columns, but the discussion of such case is necessary for explaining theory involved eccentrically loaded columns. For a column subjected purely by an axial load. The nominal load P_n that it can carry is the sum of strength steel which is $f_y A_{st}$ and the strength of concrete $0.85 f'_c(A_g - A_{st})$, where $A_g - A_{st}$ is the net concrete area, or

$$P_n = 0.85\ f'_c(A_g - A_{st}) + f_y\ A_{st} \qquad \text{Eq. 2 - 51}$$

To counter the effect of possible eccentricities, the nominal strength P_n is multiplied by0.90 for tied columns and 0.75 for spiral columns. Finally the ultimate axial load capacity of the column is Ø P_n where Ø is 0.70 for tied columns and 0.75 for spiral columns.

The axial load capacity of the tied column is given by:

$$P_u = Ø\ P_n = Ø\ 0.80\ [0.85\ f'_c(A_g - A_{st}) + f_y\ A_{st}] \qquad \text{Eq. 2 - 52}$$

Where ϕ = 0.70

A_g = gross concrete area = b x t

A_{st} = area of steel reinforcement

To counter the effect of possible eccentricities, the nominal strength P_n is multiplied by0.90 for tied columns and 0.75 for spiral columns. Finally the ultimate axial load capacity of the column is Ø P_n where Ø is 0.70 for tied columns and 0.75 for spiral columns.

These maximum load limits govern wherever the moment is small enough to keep the eccentricity under 0.10h where h is the column width parallel to the applied moment.

Limits of Reinforcement for Tied Columns

(Section 5.10.9)

l. A_{st} shall not be less than 0.01 A_g and A_{st} shall not be more than

$0.60A_g$.

ll. The minimum number of longitudinal bars is 4 for bars within rectangular or circular ties, 3 for bars within triangular ties.

Sizes and Spacing of Main Bars and Ties

l. Clear distance between longitudinal bars shall be not less than $1.5d_b$ nor 40 mm. (Section 5.7.6.3)

ll. Use 10-mm diameter ties for 32-mm bars or smaller and at least 12 mm in size for 36 mm and bundled longitudinal bars. (Section 5.7.10.5.2)

lll. Vertical spacing of ties shall be the smallest of the following: (Section 5.7.10.5.2)

1. 16 × db (db = longitudinal bar diameter)
2. 48 × tie diameter
3. Least dimension of the column

lV. Ties shall be arranged such that every corner an alternate longitudinal bar shall have lateral support provided by the corner of the tie with an included angle of not more than 135° and no bar shall be farther than 150 mm clear on each side along the tie from such a laterally supported bar. Where longitudinal bars are located around the perimeter of a circle, a complete circular tie is allowed. (Section 5.7.10.5.3)

SPIRAL COLUMN

The axial load capacity of a spiral column is given by

$$\boxed{P_u = \text{Ø}\, P_n = \text{Ø}\, 0.80\,[0.85\, f'_c(A_g - A_{st}) + f_y\, A_{st}] \qquad \text{Eq. 2 - 53}}$$

Where Ø = 0.75

This maximum load limit governs wherever the moment is small enough to keep the eccentricity under 0.05h.

Limits of Reinforcement for Spiral Columns (Section 5.10.9)

I. A_{st} shall not be less than $0.01A_g$ and A_{st} shall not be more than $0.06A_g$.

II. The minimum number of longitudinal bars is 6

Sizes and Spacing of Spirals

I. For cast-in-place construction, size of spiral shall be less than 10 mm (Section 5.7.10.4.2)

II. Clear spacing between spirals shall not exceed 75 mm, nor less than 25 mm. (Section 5.7.10.4.3)

III. Anchorage of spiral reinforcement shall be provided by 1-½ extra turns of spiral bar. (Section 5.7.10.4)

IV. Splices of spiral reinforcement shall be lap splices of 48db but not less than 300 mm or welded. (Section 5.7.10.5)

V. The percentage of spiral steel ρs is computed from the following equation

$$\rho_s = \frac{\text{volume of spiral in one loop}}{\text{voume of concrete core for a pitch s}} \qquad \text{Eq. 2 – 54}$$

$$\rho_s = \frac{4\, a_s\, (D_c - d_b)}{S\, D_c^2} \qquad \text{Eq. 2 – 55}$$

Whereas is the cross-sectional area of spiral bar, Dc is diameter of the core out to out of the spiral and db is the diameter of the spiral bar.

VI. The minimum spiral percentage is given by: (Section 5.10.9.3)

$$\rho_s = 0.45 \left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y} \qquad \text{Eq. 2 – 56}$$

Where f_y is the specified yield strength of spiral reinforcement but not more than 415 MPa.

COMPOSITE COLUMNS (Section 5.10.14)

Composite compression members include all such members reinforced longitudinally with structural steel shapes, pipe, or tubing with or without longitudinal bars. Strength of a composite member is computed for the same limiting conditions applicable to ordinary reinforced concrete members. Any axial load strength assigned to concrete of a composite member should be transferred to the concrete by members of brackets in direct bearing on the composite member concrete. All axial load strength not assigned to concrete of a composite member should be developed by direct connection to the structural steel shape, pipe, or tube.

According to Sec. 5.10.3.5.1. the design axial strength P_u of a composite member is:

$$\boxed{P_u = Ø\, P_n = Ø\, 0.85\, [0.85\, f'_c\, A_c + f_y\, A_{st} + F_y\, A_{ss}] \quad \text{Eq. 2 - 57}}$$

Where Φ = 0.75 for composite member with spiral reinforcement

Φ = 0.70 for other reinforcement

A_{st} = area of reinforcing steel of strength f_y

A_{ss} = area of structural steel shape of strength f_y

A_c = net concrete area

For evaluation of slenderness effects, radius of gyration of a composite section should not be greater than the value given by

STRUCTURAL STEEL ENCASED CONCRETE CORE (Section 5.10.14.6)

For steel pipe filled with concrete Figure 2- 24(a):

$$t_{min} = D\sqrt{\frac{f_y}{8\,E_s}} \qquad \text{Eq. 2 – 59}$$

For steel tubing filled with concrete Figure 2- 24(b):

$$t_{min} = b_1\sqrt{\frac{f_y}{3\,E_s}} \qquad \text{Eq. 2 – 60}$$

$$t_{min} = b_2\sqrt{\frac{f_y}{3\,E_s}} \qquad \text{Eq. 2 – 61}$$

Spiral Reinforcement around Structural Steel Core (Section 5.10.14.7)

A composite member with spirally reinforced concrete around a structural steel core should conform to the following:

1. Specified compressive strength of concrete fc should be not less than 17 MPa.
2. Design yield strength of structural steel core should be the specified minimum yield strength for grade of structural steel used but not to exceed 350 MPa.
3. Spiral reinforcement should conform to Sec. 5.10.9.3

4. Longitudinal bars located within the spiral should be not less than 0.01 nor more than 0.08 times net area of concrete section.
5. Longitudinal bars located within the spiral may be considered in computing A_{st} and It.

TIED REINFORCEMENT AROUND STEEL CORE (Section 5.10.14.8)

A composite member with laterally tied concrete around a structural steel core should conform to the following:

1. Specified compressive strength of concrete fc should not be less than 17 MPa.
2. Design yield strength of structural steel core should be the specified minimum yield strength for grade of structural steel used but not to exceed 350 MPa.
3. Lateral ties should extend completely around the structural steel core.
4. Lateral ties should have a diameter not less than 1/50 times the greatest side dimension of composite member, except that ties should not be smaller than Welded wire fabric of equivalent area is permitted.
5. Vertical spacing of lateral ties should not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or ½ times the least dimension of the composite member.t
6. Longitudinal bars located within the ties should be not less than 0.01 nor more than 0.08 times net area of concrete section.
7. A longitudinal bar should be located at every corner of a rectangular cross section, with other longitudinal bars

spaced not farther apart than one half the least side dimension of the composite member.

8. Longitudinal bars located within the ties may be considered in computing A_{st} for strength but not in computing it for evaluation of slenderness effects.

SLENDERNESS EFFECTS IN COLUMNS

The slenderness of columns depends on its unsupported length and the geometry of its section. As the slenderness increases, the tendency that it will buckle also increases.

To visualize the effect of slenderness, let us imagine a stick (say wire or broomstick) with the same cross-sectional area but with varying length, being compressed until it break.

According to Section 5.10.10.1 of NSCP, design of compression members should be based on forces and moments determined from analysis of the structure. Such analysis should take into account influence of axial loads and variable moment of inertia on member stiffness and fixed-end moments, effects of duration of loads. In lieu of this procedure, the slenderness effects in compression members may be evaluated in accordance with approximate procedure presented in Sec. 5.10.11.

APPROXIMATE EVALUATION OF SLENDERNESS EFFECTS (Section 5.10.11)

Unsupported Length of Compression Members

Unsupported length l_u of a compression member should be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support for that compression member. Where column capitals or haunches are present, unsupported length should be measures to the lower extremity of capital or haunch in the place considered.

Effective Length Factors (5.10.11.2.1 & 5.10.11.2.2)

For compression members braced against sideway effective length factor k shall be taken as 1 0 unless analysis shows that a lower value is justified. For compression members not braced against sideway, effective length factor k shall be determined with due consideration of effects of cracking and reinforcement on relative stiffness, and should be greater than 1.0.

Radius of Gyration

Radius of gyration r may be taken equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members, and 0.25 times the diameter for circular compression members. For other shapes, r may be computed for the gross concrete section.

For rectangular compression members:

$$\boxed{r = 0.3\,h \qquad\qquad \text{Eq. 2 – 62}}$$

Where h = overall dimension in the direction stability is being considered

For circular compression members of diameter d:

$$r = 0.25\ D \qquad \text{Eq. 2 – 62}$$

Consideration of Slenderness Effects

According to Section 5.10.11.4.1 of the Code, for compression members braced against sideway, effects of where M_{1b} is the smaller factored end moment (positive if bent in single curvature) and M_{2b} is the larger factored end moment.

For compression members not braced against sideway

Effects of slenderness may be neglected

For all compression members with k_{lu} / r > 100, an analysis as defined in Sec. 5.10.10.1 shall be made

Braced and Unbraced Frames

As a guide in judging whether a frame is braced or unbraced, the Commentary on ACI 318-83 indicates that a frame may be considered braced if the bracing elements such as shear walls. Shear trusses, or other means resisting lateral movement if a storey, have a total stiffness at least six times the sum of the stiffness of all the columns resisting lateral movement in that storey.

Alignment Charts

The ACI Committee 441 has proposed that k should be obtained from the Jackson and Moreland alignment chart as Shown in Figure 2- 27. To use this chart, a parameter Ψa for end A of column AB and similar parameter Ψb must be computed for end B. The parameter Ψ at one end of the column equals the sum of the stiffness ($\sum EI/L$) of the column meeting at that joint (including the column in question) divided by the sum of the stiffness of the beam meeting at that joint. Once Ψa and Ψb are known, k is obtained by placing a straightedge between Ψa and Ψb. The point where the straightedge crossed the middle monograph is k

$$\psi = \frac{\sum EI/L \text{ of columns}}{\sum EI/L \text{ of beams}} \qquad \text{Eq. 2 –64}$$

$\Psi = \infty$ for pinned ends and 1.0 for fixed ends

For columns for which the slenderness ratio lies between 22 and 100, and therefore the slenderness effect on load - carrying capacity must be taken into account, either an elastic analysis can be performed to evaluate the effects of lateral deflections and other effects producing secondary stresses, or an approximate method based on moment magnification may be used.

FOOTINGS

Footings are structural members used to support columns or walls and transmit their load to the underlying soils. Reinforced concrete is the most suited material for footing towers, bridges, and other structures.

Since the bearing capacity of soils is normally low (usually less than 400 kPa), and the load from a column or wall is large (usually greater than 1000 kPa), the footing spread the columns or wall pressure to the soil by providing bigger bearing area, thus reducing the bearing pressure within permissible values.

TYPES OF FOOTINGS

The common types of footing are the wall footing, isolated or single-column footing, combined footing raft or mat, and pile caps.

1. A wall footing is a continuous strip of concrete that supports a bearing wall.

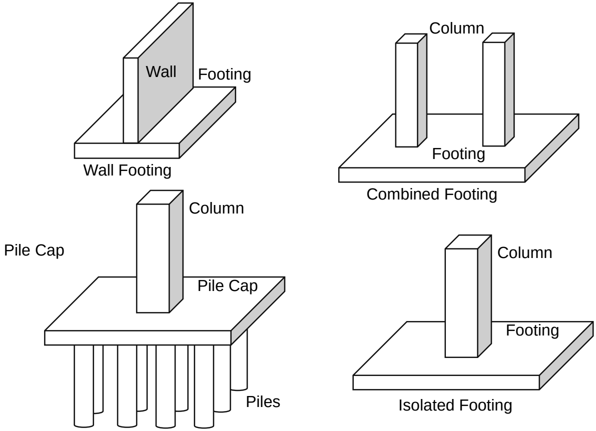
2. An isolated or single-column footing is a square, rectangular, or singular slab of concrete that supports an individual column. These are widely used for columns with light load are not closely spaced.

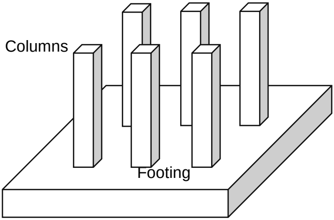
3. A combined footing is a longer rectangular slab strip that supports two or more individual columns. This type might be

economical where two heavily loaded columns are so spaced that when designed for isolated footing would run into other. Isolated footings are usually square or rectangular and, when used for columns located right at the property line, a column can be combined with an interior column to fit within the property line.

4. A floating, raft, or mat foundation is a single thick mat or slab that supports the entire structure. This kind of foundation is used where soil strength is low or where columns loads are large but where piles or caissons are not used. For these types of footing, the excavations approximately equal to the building weight.

5. Pile caps are slabs of reinforced concrete used to do distribute column loads to group of piles.





Mat or Raft Footing

PERMISSIBLE SOIL PRESSURES, q_a

The allowable soil bearing q_a capacity to be used in the design of footing can be obtained by the principles of soil mechanics through the services of a soils engineer. This can be derived on the basis of test borings, load tests, and other experimental investigation.

In the absence of soil investigation, the building code of the Philippines provide certain approximate allowable bearing pressures that can be used for the type of soil and soil conditions.

Classification of materials	**Minimum depth of footing belowad adjacent virgin ground**		**Value permissible if footing is at minimum depth**		**Increase in value for each 1 m of depth that is footing below minimum depth**		maximum value	
1	2		3		4		5	
	Meter	Feet	Kg/m²	kPa	Kg/m²	kPa	Kg/m²	kPa

Rock	0.20 0.3	1	20% ultimate crushing stength		0	0	20% ultimate crushing strength	
Compact coarse sand	0.6	2	*7,500	*75	*5,000	*50	40,000	400
Compact fine sand	0.6	2	*5,000	*50	*3,300	*33	40,000	400
Loose sand	0.9	3	*2,500	*25	*1,600	*16	15,000	150
Hard clay or sandy clay	0.6	2	20,000	200	13,300	133	40,000	400
Medium stiff clay or sandy clay	0.6	2	10,000	100	3,300	33	30,000	300
Soft sandy clay or clay	0.9	3	5,000	50	830	8.3	10,000	100
Compact inorganic sand and silt mixture	0.6	2	5,000	50	3,300	33	20,000	200
Loose inorganic sand silt mixture	0.9	3	2,500	25	1,600	16	5,000	50
Loose organic and silt mixtures and muck			0	0	0	0	0	0

These values are for footing 300 mm in width and may be increased in direct proportion to the width of the footing to a maximum of three times the designed value.

Table 2- 7: Allowable Foundation Pressure
Source: NSCP Table No. 7-B

(1) Lateral bearing and lateral sliding resistance may be combined

	Allowable Foundation Pressure kN/m² (3)	Lateral Bearing (kN/m²/m of depth) below natural	Lateral Sliding (1)	
Classes of Material (1)		Grade (4)	Coefficient (5)	Resistant KN/m² (6)
1. Massive Crystalline Bedrock	200	190	0.70	
2. Sedimentary and Foliated Rock	100	60	0.35	
3. Sandy Gravel and/or Gravel (GW & GP)	100	30		
4. Sand, Silty Sand, Clayey and Clayey Gravel and Clayey Gravel (SW, SP, Sm, SC, GM and GC)	75	25	0.25	
5. Clay, Sandy Clay, Silty Clay and Clayey Silt (CL, ML, MH, and CH)	50	15		7

(2) For soil classifications OL, OH and PT (i.e. organic clays and peat), a foundation investigation shall be required.
(3) All values of allowable soil pressure are for footing having a minimum width of 300 mm and a minimum depth of 300 mm into natural grade. Except as in Footnote (7) below, increase of 20% is allowed for each additional foot of width and/or depth to maximum value of three times the designated value.
(4) May be increased in the amount of the designated value for each additional 300 mm of depth to a maximum of 15 times the designated value. Isolated poles for uses such as flagpoles or signs or poles used to support buildings which are not adversely affected by a 12-mm motion at ground surface due to short term lateral loads may be designed using lateral bearing values equal to two times the tabulated values.
(5) Coefficient to be multiplied by the dead load.
(6) Lateral sliding resistance value to be multiplied by the contact area. In no case shall the lateral sliding resistance exceed one half the dead loads.
(7) No increase for width is allowed.

LOADS AND REACTIONS IN FOOTING

According the Code Section 5.15, the base area of footing and the number of piles may be determined from unfactored forces and moments transmitted by footing to soil or piles and permissible soil pressure or permissible pile capacity.

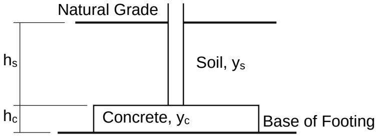
$$\boxed{\text{area of footing} = \frac{\text{unfactored load (DL+LL)}}{\text{effective soil pessure, } q_e} \quad \text{Eq. 2-74}}$$

$$\text{number of piles} = \frac{\text{unfactored load (DL+LL)}}{\text{load capacity per pile}} \qquad \text{Eq. 2 - 75}$$

Where q, is the effective soil bearing capacity and is computed as:

$$q_e = q_a - \gamma_c\, h_c - \gamma_s\, h_s \qquad \text{Eq. 2-76}$$

Where γ_c is the unit weight of concrete (usually taken as 23.54 kN/m²) h_c is the total depth of footing, γ_c is the unit weight of soil above the footing, and hc is the height of soil above the footing.



CRITICAL SECTIONS IN FOOTINGS

The critical sections for moment, shear, and development reinforcement in footings supporting a rectangular or square columns or pedestals are measured at the face of the column or pedestal. For footings supporting a circular or regular polygon shaped columns or pedestal, the Code Section 5.15.3 permits to treat these sections as square members with the same area.

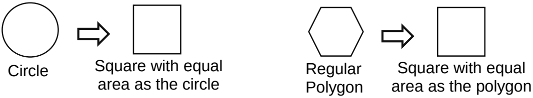


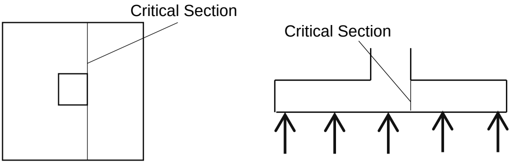
Figure 2- 29: Equivalent square sections for establishment Of critical sections

Critical Sections for Moment

Footings are similar to beams or slabs carrying the effective soil pressure as the load and the column as the support hence it is subject to moments. According to Section 5.15.4.1 the external moment on any section for a footing may be determined by passing a vertical plane to the footing, and computing the moment of the forces acting over the entire area on one side of that vertical plane.

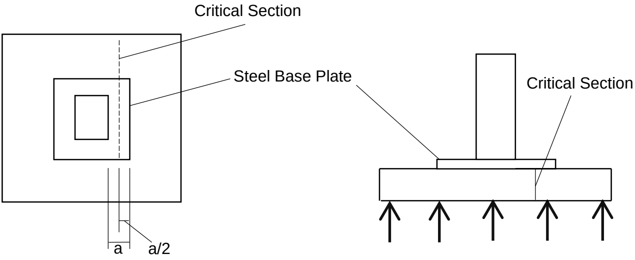
For isolated footings, the critical sections for moment are located as follows:

(a) At the face of column, pedestal, or wall for footings supporting a concrete column, pedestal, and wall.



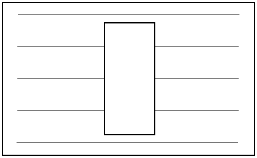
(b) Halfway between middle and edge of wall, for footing supporting a masonry wall.

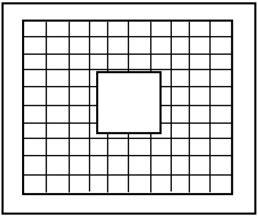
(c) Halfway between face of column and edge of steel base plate, for footing supporting a column with steel base plate.



Distribution of Flexural or Main Reinforcement

Footings may be classified as a one-way footing or two way footing. One-way footings are those, which are reinforced in one direction only while two-way footings are reinforced in two directions.


One – way footing


Two – way footing

According to Section 5.15.4 in one-way footings, and two-way square footings, reinforcement may be distributed uniformly across the entire width of footing.

In two-way rectangular footings, reinforcements may be distributed as follows:

(a) Reinforcement in long directions may be distributed uniformly across the entire width of footing.

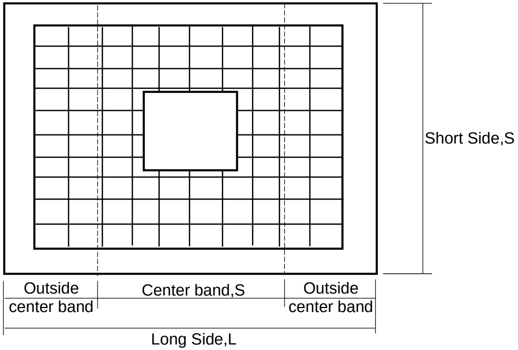


Figure 2- 30: Reinforcement distribution for two-way rectangular footing

(b) For the reinforcement in the short directions, a portion of the total reinforcement may be distributed uniformly over a bandwidth (with center on centerline of column) equal to the length of the short side of footing. The rest of the reinforcement may be distributed uniformly outside the center bandwidth of footing. The area of reinforcement in the center band is given by the formula

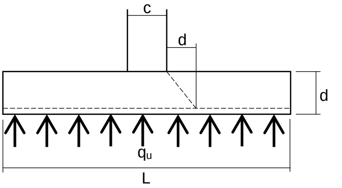
$$\frac{\text{As in center band width}}{\text{total As in short direction}} = \frac{2}{\beta + 1} \quad \text{Eq. 2-77}$$

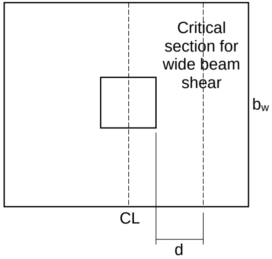
$$\beta = \frac{\text{long side of footing}}{\text{short side offooting}} \qquad \text{Eq. 2 - 78}$$

SHEAR IN FOOTINGS

The shear strength of slabs and footings in the vicinity of the columns, concentrated load, or reactions is governed by the more severe of two conditions, the beam action or one-way shear and the two-way or punching shear. In any of these two conditions, the Code requires that the maximum value of Vu if stirrups are not used $ØV_c$ is the shear strength provided by concrete.

1. Beam action (one-way), where each critical section to be investigated extends in a plane across the entire width. For this case, the slab or footing may be designed in accordance to Section 5.11.1 through Section 5.11.5. According to this section, the shear strength provided by concrete V_c may not exceed





$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w\, d \qquad \text{Eq. 2-79}$$

With reference to the figure, $V_u = q_u \times$ shaded area, where q_u is the factored soil pressure and is equal to $P_u/A_{footing}$.

2. Two-way action where each of the critical section to be investigated may be located so that its perimeter b is a minimum but need not approach closer than d/2 to:
   (a) edges or corners of columns, concentrated loads, or reactions areas or
   (b) Changes in slab thickness such as edges of capitals or drop panels.

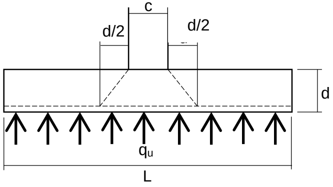
With reference to the figure, $b_o = 4(c + d)$, Vu = q u × shaded area.

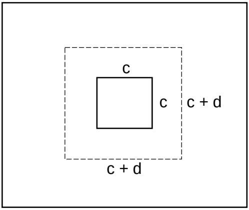
For two-way action, V_c is the smaller value of Eq. 2- 80 & Eq. 2-82.

$$V_c = \left(1 + \frac{2}{\beta_c}\right)\frac{\sqrt{f'_c}}{6}\, b_o\, d \qquad \text{Eq. 2-80}$$

$$\text{where } \beta_c = \frac{\text{long side of column}}{\text{short side of column}} \qquad \text{Eq. 2 – 81}$$

$$V_c = \frac{\sqrt{f'_c}}{3}\, b_o\, d \qquad \text{Eq. 2 - 82}$$





One-way shear will very often control the depths for rectangular footings, whereas two-way shear normally controls the depth of square footings.

Minimum Depth of Footing

According to Section 5.15.7 the depth of footing above bottom reinforcement may not be less than 150 mm for footings on soil, and 300 mm for footings on piles.

Critical Sections for Development of Reinforcement in Footings

The development of reinforcement in footings is in accordance with Section 5.12 the critical sections for development of reinforcement may be assumed at the same location as those of critical moment.

Load Transfer from Columns to Footings

All forces acting at the base of a column must be transferred into the footing. Compressive forces may be transferred directly by bearing while uplift or tensile forces must be transferred by developed reinforcing such as dowels and mechanical connectors.

At the base of the column, the permissible bearing strength of for either surfaces is Ø (0.85 fc A1), where Ø = 0.70, but it may be multiplied by $\sqrt{A\ 2/A} \leq 2$ for bearing in the footing (Section 5.10.15) where A1 is the column area and A2 is the area of the portion of the supporting footing that is geometrically similar and concentric with the columns.

Dowels

If the computed bearing force is higher than the allowable value, it is necessary to provide dowels to carry the excess force. This can also be done by extending the column bars into the footing. If the computed bearing force is less than the allowable theoretically, no dowels are needed but the code specifies a minimum value.

For cast-in-place columns and pedestal, the area of reinforcement across interface shall not be less than 0.005 times the gross area of the column or pedestal, and at footings and 36-mm longitudinal bars in compression only may be lap spliced with dowels to provide the required reinforcement. Dowels may not be larger than 32 mm bar and may extend into column a distance not less than the development length of 36 mm bars or the splice length of the dowel, whichever is greater, and into the footing a distance not less than the development length of the dowels (Section 5.15.8.2.3)

COMBINED FOOTINGS

Combined footings support more than one column. One situation where these footings may be used is when the columns are close together so that isolated or individual footing would run into each other. Another situation is when the column is very near the property line. A trapezoidal footing or strap (T) footings may also be used is the two adjacent column are very near the property line.

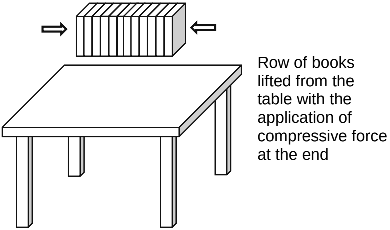
In any of these shapes, it is very important to let the centroid of the footing coincide with the centroid of the combined column loads. In this manner the bearing pressure underneath the footing would be uniform and it prevents uneven settlement.

PRESTRESSED CONCRETE

Prestressed concrete are those in in which cracking and tensile forces are greatly reduced or eliminated by the imposition of internal stress that are of opposite character to those that will be caused by the service or working loads.

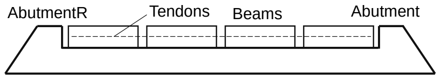
The materials used in prestressed concrete are concrete and high strength steels also known tendons. The concrete to be used have higher strength than that used for reinforced concrete members.

Analogy of Prestressing



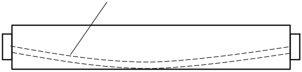
Methods of Prestressing

There are two general methods of prestressing, these are pretensioning and posttensioning. In pretensioning, tendons were tensioned before the concrete was placed. After the concrete had hardened sufficiently, the tendons are cut and prestress force is transmitted to concrete by bond. This method is well suited for mass production where the tendons can run to several meters long across several beams in the casting bed, as shown in the figure below.

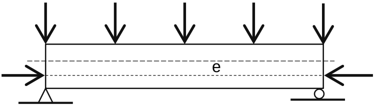


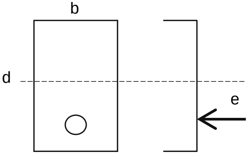
In posttensioning, the tendons are tensioned after the concrete is placed and has gained the required strength. The tendons are placed inside hallow ducts or tunes located in the form. When the concrete has hardened, the tendons are stretched and mechanically attached to end anchorage. In this method, the prestress force is transferred to the concrete by end bearing.

Hallow ducts where tendons are placed



Stress Calculation





The stresses to be considered in prestressed concrete are those due to (a) the direct compressive force by the tendons (b) the moment due to the eccentricity of the prestress and (c) the flexural stress due to loadings. The resultant stress at any section is the algebraic sum of these stresses at that section with compressive stress being negative and tensile stress positive.

General Equation:

$$f = -\frac{P}{A} \pm \frac{Pec}{I} \pm \frac{Mc}{I} \qquad \text{Eq. 2-83}$$

For rectangular section:

$$f = -\frac{P}{bd} \pm \frac{6Pe}{bd^2} \pm \frac{6M}{bd^2} \qquad \text{Eq. 2-84}$$

Where P = prestressing force

e = eccentricity

M = moment due to loading

I = moment of inertia of the gross section

Rule of Sign

The first term of the equation is always negative (compressive).

For the second term of the equation use negative (-) to get the stress at the bottom and positive (+) to get the stress at the top.

For the third term, use the positive (+) sigh if the bending causes tension in the fiber and negative (-) if the bending causes compression in the fiber.

Loss of Prestress

The immediate prestressing force applied on concrete is called initial stress. The stresses, however, reduces with losses must be considered to determine the effective prestress f_{se}. According to Section 5.18.6.1 the following losses must be considered.

1. Anchorage seating loss - When the jacks are released and the prestress forces transferred to the end anchorage system, a little slippage of the tendon occurs. This slippage shortens the tendons thus reduces its stress.
2. Elastic shortening of concrete - When tendons are cut for a pretensioned member, the prestress force is transferred to the concrete, with the result that the concrete is put in compression and shortens. This causes the tendons to shorten also thus losses some stress. The loss in the stress can be calculated by the formula.

$$\Delta f_S = n\, f_C = \frac{n\, P_O}{A_g} \qquad \text{Eq. 2-85}$$

Where Δf_s is the loss of prestress, fc is the stress in concrete after transfer of stresses from the cables, n is the modular ratio which is equal to E_s/E_c, P_o is the initial cable stress, and Ag is the gross concrete area.

3. Creep of concrete - The gradual deformation of concrete under stress causing a reduction of the length of tendon.

   The loss in cable stress due to creep can be determined by multiplying the creep coefficient C_t by nf_c.

$\Delta f_S = C_1 \, n \, f_C$	Eq. 2-86

   The value of C_t = 2.0 is recommended for pretensioned section and 1.6 for posttensioned ones, f_c is the stress in concrete adjacent to the centroid tendons due to the initial prestress (-P/A) and due to the permanent dead loads which are applied to the members after prestressing (-Pec/I) where c is measured from the centroid of the selection to the centroid of the tendons.

4. Shrinkage of concrete – Concrete shrinks during setting and hardening. The amount of shrinkage that occurs in concrete varies from almost zero to 0.0005 mm/mm with an average value of about 0.0003 mm/mm. The shrinkage loss is approximately 7% in pretensioned sections and 6% for the posttensioned ones. The loss in the prestress due to shrinkage is equal to ε_{sh} is given by the formula

$\varepsilon_{sh} = 0.00055\,(1 - 0.06\,V/S)\,(1.5 - 0.15\,H)$	Eq. 2-87

Where V/S is the volume to surface ratio and H is the relative humidity correction

5. Relaxation of tendon stress - This refers to the creep of tendons due to permanent stress.

6. Friction loss due to intended or unintended curvature in post-tensioning tendons - This refers to the friction loss between the tendon and the surrounding materials this loss are due to the so-called length and curvature effects. The length effect or wobble effect is the friction that would have existed if the cable had been straight and not curved. The curvature effect is caused by the coefficient of friction between the materials caused by the pressure on the concrete from the tendons.

UPDATES FROM NSCP 2001 (C101-01)

Factor β_1

410.3.7.3 Factor β, shall be taken as 0.85 for concrete strengths above 30 MPa, β, shall be reduced continuously at a rate of 0.05 for each 7 MPa of strength in excess of 30 MPa but β,shall not be taken less than 0.65.

For $f_C \leq 30$ MPa, $\beta_1 = 0.85$ Eq. 2-88

For $f_C > 30$ MPa, $\beta_1 = 0.85 - \frac{0.05}{7}(f_C - 30) \geq 0.$ Eq. 2-89

Minimum Reinforcement of flexural Members

410.6.1 A every section a flexural member where tensile reinforcement is required by analysis, the area A_s provided shall not be less than that given by

$$A_{S\,min} = \frac{\sqrt{f'_C}}{4f_y}\, b_W\, d \qquad \text{Eq. 2-90}$$

$$\text{and not less than} \quad \frac{1.4\, b_W\, d}{f_y} \qquad \text{Eq. 2-91}$$

410.6.2 For statically determinate T-section with flange in tension, the area $A_{S\,min}$ shall be equal to or greater than the smaller value given either by:

$$A_{S\,min} = \frac{\sqrt{f'_C}}{2f_y}\, b_W\, d \qquad \text{Eq. 2-92}$$

or Eq. 2-90 with b_W set equal to the width of the flange

410.6.3 The requirements of Sections 410.6.1 and 410.6.2 need not to be applied if at every section the area of the tensile reinforcement is at least one-third greater than that required by the analysis

410.6.4 For structural slabs and footings of uniform thickness, the minimum area of tensile reinforcement in the direction of span shall be the same as that required by Section 407.13

(Shrinkage and Temperature Reinforcement). Maximum Spacing of this reinforcement shall not exceed three times the thickness and 450 mm

DESIGN FOR TORSION

A_{CP} = area enclosed by outside perimeter of concrete cross section mm²

A_l = total area of longitudinal reinforcement to resist torsion mm²
A_o = gross area enclosed by shear flow, mm²

A_{oh} = area enclosed by center line of the outermost closed transverse torsional reinforcement, mm²

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, mm²

f_{pc} = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross-section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange, MPa

f_{yl} = yield strength of longitudinal torsional reinforcement MPa

f_y = yield strength of closed transverse torsional reinforcement, MPa

h = overall thickness of member, mm

P_{cp} = outside perimeter of the concrete cross-section mm

P_h = perimeter of centerline of outermost closed transverse torsional reinforcement, mm

S = spacing of shear or torsion reinforcement indirection parallel to longitudinal reinforcement, mm

θ **=** angle of compression diagonals in truss analogy for torsion

411.7.1 It shall be permitted to neglect torsion effects when the factored torsional moment T_u is less than.

1. For non-prestressed members:

$$\frac{\phi\sqrt{f'_C}}{12}\left(\frac{A_{cp}{}^2}{P_{cp}}\right) \qquad \text{Eq. 2-93}$$

2. For prestressed members:

$$\frac{\phi\sqrt{f'_C}}{12}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)\sqrt{1+\frac{3f_{pc}}{\sqrt{f'_C}}} \qquad \text{Eq. 2-94}$$

For members cast monolithically with a slab, the overhanging flange width used in computing a A_{cp} and P_{cp} shall conform to Section 413.3.4

411.7.2 Calculation of Factored Torsional Moment T_u.

411.7.2.1 If the factored torsional moment T_u in a member is required to maintain equilibrium and exceeds the minimum value given in section 411.7.1, the member shall be designed to carry the torsional moment in accordance with sections 41 1.7.3, through 411.7.6.

411.7.2.2 In a statically indeterminate structure where reduction of the torsional moment in a member can occur due to the redistribution of internal forces upon cracking, the maximum factored torsional moment Tu shall be permitted to be reduced to

1. For non-prestressed members. At the section described in Section 411.7.2.4

$$\frac{\phi\sqrt{f'_C}}{3}\left(\frac{A_{cp}^{2}}{P_{cp}}\right) \qquad \text{Eq. 2-95}$$

2. For prestressed members. At the sections decribed is Section 411.7.2.5

$$\frac{\phi\sqrt{f'_C}}{3}\left(\frac{A_{cp}^{2}}{P_{cp}}\right)\sqrt{1+\frac{3f_{pc}}{\sqrt{f'_C}}} \qquad \text{Eq. 2-96}$$

In such a case, the correspondingly redistributed bending moments and shears in the adjoining members shall be used in the design of those members

411.7.2.3 Unless determined by a more exact analysis, it shall be permitted to take the torsional loading from slab as uniformly distributed along the member

411.7.2.4 In non-prestressed members, sections located less than a distance d from the face of a support shall be designed for not less than the torsion T_u computed at a distance d. if a concentrated torque occurs within this distance the critical section for design shall be at the face of the support.

411.7.2.5 In prestressed members, section located less than a distance h/2 from the face of a support shall be designed for not less than the torsion T u computed at a distance h/2 if a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support

411.7.3 Torsional Moment Strength

411.7.3.1 The cross-sectional dimensions hall be such that:

1. For solid sections:

$$\boxed{\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7\, A_{oh}^2}\right)} \leq \phi \left(\frac{V_c}{b_w d} + \frac{2\sqrt{f'_C}}{3}\right) \qquad \text{Eq. 2-97}}$$

2. For hollow sections:

$$\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7\, A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + \frac{2\sqrt{f'_c}}{3}\right) \qquad \text{Eq. 2-98}$$

411.7.3.2 If the wall thickness varies around the perimeter of a hollow section, Eq. 2- 98 shall be evaluated at the location where the left-hand side of Eq. 2- 98 is a maximum.

411.7.3.3 If the wall thickness is less than A_{oh}/P_h, the second term in Eq. 2- 98 shall be taken as:

$$\left(\frac{T_u}{1.7\, A_{oh} t}\right) \qquad \text{Eq. 2 - 99}$$

Where t is thickness of the wall of the hollow section at the location where the stresses are being checked.

411.7.3.4 Design yield strength of non-prestresses torsion reinforcement shall not exceed 415 MPa.

411.7.3.5 The reinforcement required for torsion shall be determined from:

$$T_u \leq \phi\, T_n \qquad \text{Eq. 2-100}$$

411.7.3.6 The transverse reinforcement for torsion shall be designed using:

$$T_n = \frac{2A_o A_t f_{yv}}{s} \cot^2\theta \qquad \text{Eq. 2-101}$$

Where A_o shall be determined by analysis except that it shall be permitted to take A_o equal to 0.85Aoh; θ shall not be taken smaller than 30 degrees nor larger than 60 degrees. It shall be permitted to take θ equal to:

1. 45° for non-prestressed members or members with less prestress than in Item 2 below,
2. 37.5° or prestressed members with an effective prestressed force not less than 40 percent of the tensile strength of the longitudinal reinforcement.

411.7.3.7 The additional longitudinal reinforcement required for torsion shall not be less than:

$$A_t = p_h \frac{A_t}{s} \frac{f_{yv}}{f_{yl}} \cot^2\theta \qquad \text{Eq. 2-102}$$

Where θ shall be the same value used in Eq. 2- 101 and A_t/s shall be taken as the amount computed from Eq. 2- 101 not modified in accordance with section 411.7.5.2 or 411.7.5.3

411.7.3.8 Reinforcement required for torsion shall be added to that shear, moment and axial force that act in combination with the torsion. The most restrictive requirements for reinforcement spacing and placement must be met.

411.7.3.9 It shall be permitted to reduce the area of longitudinal torsion reinforcement in the flexural compression zone by an amount equal to M_u/ (0.9 d f_{yl}), where Mu is the factored moment acting the section in combination with T_u except that the reinforcement provided shall be less than required by section 411.7.5.3 or 411.7.6.2

411.7.3.10 In Prestressed Beam:

1. The total longitudinal reinforcement including bending moment at that section shall resist the factored bending moment at that section plus a_b additional concentric longitudinal tensile force equal to A_l f_{yl}, based on the factored torsion at that section, and

2. The spacing of the longitudinal reinforcement including tendons shall satisfy the requirements in section 411.7.6.2

411.7.3.11 In prestressed beams, it shall be permitted to reduce the area of longitudinal torsional reinforcement on below that required by section 411.7.3.10 in accordance with section 411.7.3.9

411.7.4 Details of torsional reinforcements

411.7.4.1 Torsion reinforcement shall consist of longitudinal bars or tendons and one or more of the following:

1 .Closed stirrups or close ties, perpendicular to the axis of the member, or

2. A close cage of welded wire fabric with transverse

wires perpendicular to the axis of the member, or
3. Non-prestressed beams, spiral reinforcement.

411.7.4.2 Transverse torsional reinforcement shall be anchored by one of the following:

1. A 135 degree standard hook around a longitudinal bar, or
2. According to section 412.14.2.1, 412.14.2.2 or 412.14.2.3 in regions where the concrete surrounding the anchorage is restrained against spalling by a flange or slab or similar member

411.7.4.3 Longitudinal torsion reinforcement shall be developed at both ends.

411.7.4.4 For hollow sections in torsion, the distance measured from the centerline of the transverse torsional reinforcements to the inside face of the wall of the hollow section shall not be less than 0.5 A_{oh}/p_h

411.7.5 Minimum Torsion Reinforcement

411.7.5.1. A minimum area of torsion reinforcement shall be provided in all regions where the factored torsional moment 1; Exceeds the values specified in section 411.7.1.
411.7.5.2. Where the torsional reinforcements are required by section 411.7.5.1 the minimum area of transverse closed stirrups shall be computed by:

$$A_v + 2A_t = \frac{1\, b_w\, s}{3\, f_v} \qquad \text{Eq. 2-103}$$

411.7.5.3 Where torsional reinforcement is required by section 411.7.5.1 the minimum total area of longitudinal torsional reinforcement shall be computed by:

$$A_{tmin} = \frac{5\sqrt{f'_c}A_{cp}}{12\ f_{vt}} - \left(\frac{A_t}{s}\right)\frac{f_{yv}}{f_{yt}}P_n \qquad \text{Eq. 2-103}$$

Where A_t/s shall not be taken less than $1/6\ b_w/f_{yv}$.

411.7.6 Spacing of Torsion Reinforcement.

411.7.6.1 The spacing of transverse torsion reinforcement shall not exceed the smaller of $P_h/8$ or 300 mm.

411.7.6.2 The longitudinal reinforcement required for torsion shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 300 mrn. The longitudinal bars or tendons shall be inside the stirrups. There shall be at least one longitudinal or tendon in each comer of the stirrups. Bars shall have a diameter at least 1/24 of the stirrup spacing but not less than a 10 mm diameter bar.

411.7.6.3 Torsion reinforcement shall be provided for a distance of at least (b_t + d) beyond the point theoretically required.

DEVELOPMENT AND SPLICES OF REINFORCEMENT

Notations

A_b = area of an individual bar, mm²

A_s = area of non-prestresses tension reinforcement, mm²

A_t =total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, mm²

A_v = area of shear reinforcement within a distance s, mm²

A_w = area of an individual wire to be developed or spliced, mm²
a =depth pf equivalent rectangular stress block as defined in section 410.3.7.1, mm

b_w =web width, or diameter of singular section, mm

c = spacing or cover dimension, mm. See section 412.3.4

d= distance from extreme compression fiber to centroid of tension reinforcement, mm

d_b=nominal diameter of bar, wire or prestressing strand, mm.

f_c = specified compressive strength of concrete, MPa

$\sqrt{f_c}$ = square root specified compressive strength of concrete, MPa

f_{ct}= average splitting tensile strength of lightweight aggregate concrete, MPa

f_{ps}= stress in prestressed reinforcement at nominal strength, MPa

f_{sc} =effective stress in prestressed reinforcement(after allowance for all prestressed losses), MPa

f_y = specified yield strength of transverse reinforcement, MPa h=overall thickness of member, mm

K_{tr}= transverse reinforcement index = A_{tr} f_{yt}/10sn

l_a= additional embedment length at support or at point of inflection, mm

l_d=development length, mm

l_d = l_{db} multiplied by applicable modification factors

l_{db}= basic development length , mm

l_{dh} =development length of standard hook in tension, measured from critical section to outside end of hook [straight embedment length between critical section and start of hook (point of tangency) plus radius of bend and one bar diameter, mm

l_{dh} = l_{hb} times applicable modification factors

l_{hb} = basic development length of standard hook in tension, mm

M_n= nominal moment strength at section, Newton meter

$M_n = A_s f_y (d-a/2)$

N= number of bars in a layer being spliced or developed at a critical section

n = number of bars or wires being spliced or developed along the plane of splitting

s = maximum center to center spacing of transverse reinforcement within l_d, mm

Sw = spacing of wire to be developed or spliced, mm

V_u = factored shear force at section.

α = reinforcement location factor. See Section 412.3.4.

β = coating factor. See Section 412.3.4.

β_b = ratio of area of reinforcement cut off to total area of tension reinforcement at section

γ = reinforcement size factor. See section 412.3.4

λ = lightweight aggregate concrete factor. See Section 412.3.4.

412.2 Development of Reinforcement - General

412.2.1 Calculated tension or structural concrete members shall be developed on each side of that section by embedment length, hook or mechanical device, or a combination thereof hook shall not be used to develop bars in compression.

412.2.2 The values of fc used in Section 412 shall not exceed 8.0 MPa.

412.3 Development of deformed Bard and Deformed Wire in Tension

412.3.1 Development length, l_d, in terms of diameter, d_b, for deformed bars and deformed wire in tension shall be determined from either Section 412.3.2 or 412.3.3 but l_d shall not be less than 300 mm.

412.3.2 For deformed bars or deformed wire, l_d/d_b shall be as follows:

	20 mm diameter and smaller bars and deformed wires	25 mm diameter and larger bars
Clear spacing of bars being developed or spliced not less than d_b clear cover not less than d_b and stirrups or ties throughout l_d not less than the code minimum or Clear spacing of	$\frac{l_d}{d_b} = \frac{12\, f_y\, \alpha\, \beta\, \lambda}{25\sqrt{f'_c}}$	$\frac{l_d}{d_b} = \frac{3\, f_y\, \alpha\, \beta\, \lambda}{5\sqrt{f'_c}}$

bars being developed or spliced not less than d_b		
Other cases	$\frac{l_d}{d_b} = \frac{18\, f_y\, \alpha\, \beta\, \lambda}{25 \sqrt{f'_c}}$	$\frac{l_d}{d_b} = \frac{9\, f_y\, \alpha\, \beta\, \lambda}{10 \sqrt{f'_c}}$

412.3.3 For deformed bars or deformed wire, ld/db shall be:

$$\frac{l_d}{d_b} = \frac{9\, f_y\, \alpha\, \beta\, \gamma\, \lambda}{10 \sqrt{f'_c} \left(\frac{c + k_{tr}}{d_b}\right)} \qquad \text{Eq. 2-105}$$

In which the term $(c + k_{tr})/d_b$ shall not be taken greater than 2.5

412.3.4 The factors for use in the expressions for development of deformed bars and deformed wires in tension in Sections 412.1 through 412.20 are as follows:

Reinforcement factor, α	
For horizontal reinforcement so placed that more than 300 mm of fresh concrete is cast in the member below the development length or splice	$\alpha = 1.3$
For other reinforcement	$\alpha = 1.0$
Coating factor, β	
For epoxy-coated bars or wires with cover less than 3db or clear spacing less than 6db	$\beta = 1.5$
All other epoxy-coated bars or	

wires	
For uncoated reinforcement	$\beta = 1.0$
However, the product $\alpha\,\beta$ need not be taken greater than 1.7	
Reinforcement size factor, γ	
For 20 mm diameter and smaller bars and deformed wires	$\gamma = 0.8$
For 25 mm diameter and larger bars	$\gamma = 1.0$
Lightweight aggregate concrete factor, λ	
When lightweight aggregate concrete is used, however, when f_{ct} is specified, λ shall be permitted to be $f_c/1.8f_{ct}$ but not less than 1.0	$\lambda = 1.3$
When normal weight concrete is used	$\lambda = 1.0$

C = spacing or cover dimension, millimeters. Use the smaller or either the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bars being developed.

K_{tr} = transverse reinforcement index = $A_s\, f_{yt}/10\ s\ n$

Where:
A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, square millimeters.

f_{yt} = specified yield strength of transverse reinforcement square millimeters.

s = maximum center-to-center spacing of transverse reinforcement within ld, millimeters.
N = number of bars or wires being developed along the plane of splitting

it shall be permitted to use K_{tr} = 0 as a design simplification even if transverse reinforcement is present.

412.3.5 **Excess reinforcement.** Reduction in development length shall be permitted where reinforcement in flexural member is in excess of that required by analysis except where anchorage or development for h is specifically required or the reinforcement is designed under provisions of Selection 421.2.1.4 …………… [(A_s required)/(A_s provided)]

412.4 Development of Deformed Bars in Compression

412.4.1 Development length l_{db} and applicable modification factors as defined in this section. But l_d shall not less than 200 mm

412.4.2 Basic development length l_{db} shall be

$$l_{db} = \frac{f_y \, d_b}{4\sqrt{f'_c}} \qquad \text{Eq. 2-106}$$

412.4.3 Basic development length l_{db} shall be permitted to be multiplied by applicable factors for

412.4.3.1 Excess reinforcement.

Reinforcement in excess of that required by analysis …………… [(A_s required)/(A_s provided)]

412.4.3.2 Spirals and Ties Reinforcement enclosed within spiral reinforcement not less than 10 mm diameter and not more than 100 mm pitch or within 12 mm diameter ties in conformance with section 407.11.5 and spaced not more than 100 mm on center……………………………………………..0.75

ALTERNATE DESIGN METHOD (WORKING STRENGTH)

Notations

Ag = gross of section, mm^2
A_v = area of shear reinforcement within a distance s, mm^2
A_1 = loaded area
A_2 =maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area
b_o = perimeter of critical section for slabs and footings, mm
b_w = web width, or diameter of circular section, mm
d = distance from extreme compression fiber to centroid of tension reinforcement, mm
E_c = modulus of elasticity of concrete, MPa.
E_s = modulus of elasticity of reinforcement, MPa
F_c = specified compressive strength of concrete, MPa
$\sqrt{f'_c}$ = square root of specified compressive strength concrete, MPa
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, MPa
f_s = permissible tensile stress in reinforcement, MPa
f_y = specified yield strength of reinforcement, MPa
M = design moment
n = modular ratio of elasticity E^δ/Ec
N = design axial load normal to cross section occurring simultaneously with V; to be taken as positive for compression, negative for tension, and to include effects of tension due to

creep and shrinkage

s = spacing of shear reinforcement in direction parallel to longitudinal reinforcement. mm

v = design shear stress

v_c = permissible shear stress carried by concrete, MPa

V_h = permissible horizontal shear stress, MPa

V = design shear force at section

α = angle between inclined stirrups and longitudinal axis of member

β_c = ratio of long side to short side of concentrated load or reaction area

ρ_w = ratio of tension reinforcement = $A_s/b_w d$

ϕ = strength reduction factor

424.2 Scope

424.2.1 Non-prestressed reinforced concrete members shall be permitted to be designed using service loads (without load factors) and permissible service load stresses in accordance with provisions of Section 424

424.2.2 For design of members not covered by Section 424, Appropriate provisions of this code shall apply.

424.2.3 All applicable provisions of this code for nonprestressed concrete, except Section 408.5, shall apply to members designed by the Alternate Design Method.

424.2.4 Flexural members shall meet requirements for deflection control in Section 409.6, and requirements of Sections 410.5 through 410.8 of this code.

424.3 General

424.3.1 Load factors and strength reduction factors ϕ shall be taken as unity for members designed by the Alternate Design Method.

424.3.2 It shall be permitted to proportion members for 75 percent of capabilities required by other parts of Section 424 when considering wind or earthquake forces combined with other loads, provided the resulting section is not less than that required for the combination of dead and live load.

424.3.3 When dead load reduces effects of other loads, members shall be designed for 85 percent of dead load in combination with the other loads.

424.4 Permissible Service Load Stresses

424.1 Stresses in concrete shall not exceed the following:

1. Flexure:

® Extreme fiber stress in compression………………. $0.45 f^1 c$

2. Shear:

® Beams and one way slabs and footings:

- Shear carried by concrete, Vc …………. $0.38\sqrt{f^1} c$
- Maximum shear carried by concrete plus shear reinforcement, Vc ……………………….. $0.38\sqrt{f^1} c$

® Joists:

- Shear carried by concrete, Vc ………….. $0.09\sqrt{f^1} c$

® Two-way slabs and footings:

- Shear carried by concrete, Vc

$$\frac{1}{12}\left(1+\frac{2}{\beta c}\right)\sqrt{f'c}$$

But not greater than $\frac{1}{6}\sqrt{f'c}$

3. Bearing on loaded area ………………. 0.3 f' c

424.4.2 Tensile stress in reinforcement shall not exceed the Following:

1. Grade 275 ……………………………. 140MPa
2. Grade 425 reinforcement or greater and Welded wire fabric (plain or deformed) ……….. 170MPa
3. For flexural reinforcement, 10mm or less In diameter, in one way slabs of not More than 4 m span ………………….. $0.50f_y$

424.5 Development and Splices of Reinforcement

424.5.1 Development and splices of reinforcement shall be as required in Section 412 of this Chapter.

424.5.2 In satisfying requirements of Section 412.12.3, M_n shall be taken as computed moment capacity assuming all positive moment tension reinforcement at the section to be stressed to the permissible tensile stress f_s and V_u shall be taken as unfactored shear force at the section.

424.6 Flexure

For investigation of stresses at service loads, straight-line theory (for flexure) shall be used with the following assumptions:

424.6.1 Strains vary linearly as the distance from the neutral axis, except for deep flexural members with overall depth- span ratios greater than 2/5 for continuous spans and 4/5 for simple spans, a nonlinear distribution of strain shall be considered.
424.6.2 Stress-strain relationship of concrete is a straight line under service loads within permissible service load stresses.

424.6.3 In reinforced concrete members, concrete resists no tension.

424.6.4 It shall be permitted to take the modular ratio, n =
E_s/E_c as the nearest whole number (but not less than 6).
Except in calculations for deflections, value of n for lightweight concrete shall be assumed to be the same as for normal weight concrete of the same strength.

424.6 5 In doubly reinforced flexural members, an effective modular ratio of $2E_s/E_c$ as shall be used to transform compression reinforcement for stress computations.
Compressive stress in such reinforcement shall not exceed permissible tensile stress.

424.7 Compression Members With or Without Flexure

424.7.1 Combined flexure and axial load capacity of compression members shall be taken as 40% of that computed on accordance with provisions on Section 410 if this Chapter.

424.7.2 Slenderness effects shall be included according to requirements of Sections 410.10 through 410.13 in
Equations (410-10) and (410-19) the rem P_u shall be
Replaced by 2.5 times the design axial load, and the factor
0.75 shall be taken equal to 1.0.

424.7.3 Walls shall be designed in accordance with Section 414 of this Chapter with flexure and axial load capacities taken as 40 percent of that computed using Section 414. In Equation (414-1), ϕ shall be taken equal to 1.0.

424.8 Shear and Torsion

$$v = \frac{V}{b_w\, d} \qquad \text{Eq. 2-107}$$

424.8.1 Design shear stress v shall be computed by

Where V is design shear force at section considered.

424.8.2 When the reaction, in direction of applied shear, introduces compression into the end regions of a member, sections located less than a distance d from face of support shall be permitted to be designed for the same shear as that computed at a distance d.

424.8.3 Whenever applicable, effects of torsion, in accordance with provisions of Section 411 of this Chapter, shall be added. Shear and torsional moment strengths. Provided by concrete and limiting maximum strengths for torsion shall be taken as 55 percent of the values given in Section 411.

424.8.4 Shear stress carried by concrete

424.8.4.1 For members subject to shear and flexure only, shear stress carried by concrete V_c shall not exceed
$0.09\ \sqrt{f^1c}$ unless a more detailed calculation is made in

accordance with Section 424.7.4.4.

424.8.4.2 For members subject to axial compression.
Shear stress carried by concrete V_c, shall not exceed 0.09 √f¹c unless a more detailed calculation is made in accordance with 424.7.4.5.

424.8.4.3 For members subject to significant axial tension,
Shear reinforcement shall be designed to carry total shear, unless a more detailed calculation is made using where N is negative for tension. Quantity N/Ag shall be expressed in MPa.

$$V_c = 0.09\,(1+ 0.6\,N/A_g)\sqrt{f'_c} \qquad \text{Eq. 2-108}$$

424.8.4.4 For members subject to shear and flexure only.
It shall be permitted to compute V_c by

$$V_c = 0.85\sqrt{f'_c} + 0.9\,\rho_w V_d/M \qquad \text{Eq. 2-109}$$

But V_c shall not exceed 0. 14 √f¹c. Quantity V_d/m shall not be taken greater than 1.0. Where M is design moment occurring simultaneously with V at section considered.

$$V_c = 0.09\,(1+ 0.09\,N/A_g)\sqrt{f'_c} \qquad \text{Eq. 2-110}$$

424.8.4.5 For members subject to axial compression, it
Shall be permitted to compute V_c by quantity N/A_g shall be expressed in MPa.

424.8.4.6 Shear stresses carried by concrete V_c apply to normal weight concrete. When lightweight aggregate concrete is used, one of the following modifications shall apply:

1. When f_{ct} is specified and concrete is proportioned in accordance with Section 405.3, $f_{cr}/6.7$ shall not exceed $\sqrt{f^1c}$
2. When f_a is not specified the valued of following shall be multiplied by 0.75 for “all-lightweight” concrete and by 0.85 for “sand-lightweight” concrete. Linear interpolation shall be permitted when partial sand replacement is used.

424.8.4.7 In determining shear stress carried by concrete V_c. Whenever applicable, effects of axial tension due to Creep and shrinkage in restrained members shall be Included and it shall be permitted to include effects of Inclined flexural compression in variable-depth members.

424.8.5 Shear Stress Carried by Shear Reinforcement

424.8.5.1 Types of shear reinforcement
Shear reinforcement shall consist of one of the following:

1. Stirrups perpendicular to axis of member;
2. Welded wire fabric with wires located perpendicular to axis of member making an angle of 45 degrees or more with longitudinal tension reinforcement;

3. Longitudinal reinforcement with bent portion making an angle of 30 degrees or more with longitudinal tension reinforcement;
4. Combinations of stirrups and bent longitudinal reinforcement;
5. Spirals.

424.8.5.2 Design yield strength of shear reinforcement shall not exceed 415 MPa

424.8.5.3 Stirrups and other bars or wired used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according, to Section 412.14 of this Chapter to develop design yield strength of reinforcement.

424.8.5.4 Spacing limits for shear reinforcement

424.8.5.4.1 Spacing of shear reinforcement placed perpendicular to axis of member shall not exceed d/2 nor 600 mm.

424.8.5.4.2 Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45-degree line, extending toward the reaction from mid-depth of member (d/2) to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement.

424.8.5.4.3 When (v – vc) exceeds $\frac{1}{6}$ $\sqrt{f^1c}$ maximum spacing given in Sections 424.7.5.4.1 and 424.7.5.4.2 shall be reduced by one-half

424.8.5.5 Minimum shear reinforcement

424.8.5.5.1 A minimum area of shear reinforcement shall be provided in all reinforced concrete flexural members where design shear stress V is greater than one-half the permissible shear stress V_c carried by concrete, except;

1. Slabs and footings;
2. Concrete joist construction defined by Section 408.12 f this Chapter;
3. Beam with total depth .not greater that 250mm 2.5 times thickness of flange or one-half the width of web, whichever is greatest.

424.8.5.5.2 Minimum shear reinforcement requirements of Section 424.8.5.5.1 shall be permitted to be waived if shown by test that required ultimate flexural and shear strength can be developed when shear reinforcement is omitted.

424.8.5.5.3 Where shear reinforcement is required by section 424.8.5.5.1 or by analysis, minimum area of shear reinforcement shall be computed by where b_w and s are in mm.

$$A_y = \frac{b_w\, s}{3\, f_y} \qquad \text{Eq. 2-111}$$

424.8.5.6 Design of shear reinforcement

424.8.5.6.1 Where design shear stress V exceeds shear stress carried by concrete V_c, shear reinforcement shall be provided in accordance with Sections 424.8.5.6.2 through 424.8.5.6.8.

424.8.5.6.2 When shear reinforcement perpendicular to axis of member is used.

$$A_v = \frac{(v - v_c)\, b_w\, s}{f_y} \qquad \text{Eq. 2 - 112}$$

424.8.5.6.3 When inclined stirrups are used as shear reinforcement,

$$A_v = \frac{(v - v_c)\, b_w\, s}{f_y\,(\sin\alpha + \cos\alpha)} \qquad \text{Eq. 2 - 113}$$

424.8.5.6.4 When shear reinforcement consists of a single bar or a single group of parallel bus, all bent up at the same distance from support.

$$A_v = \frac{(v - v_c)\, b_w\, d}{f_s \sin\alpha} \qquad \text{Eq. 2 - 114}$$

Where $(v - v_c)$ shall not exceed $\frac{1}{8}\sqrt{f^1c}$.

424.8.5.6.5 When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support required are shall be computed by Eq. 2-113.

424.8.5.6.6 Only the center of three- quarters of the inclined portion of any longitudinal bent bar shall be considered effective for shear reinforcement.

424.8.5.6.7 When more than one type of shear reinforcement is used to reinforce the same portion of a member, required area shall be computed as the sum of the various types separately. In such computations, v_c shall be included only once.

424.8.5.6.8 Value of $(v – v_c)$ shall not exceed $\frac{3}{8}\sqrt{f^1c}$

424.8.6 Shear Friction

Where it is appropriate to consider shear transfer across a given plane, such as existing or potential crack, an interface between dissimilar materials, or an interface between two concrete cast at different times, shear-friction provisions of Section 411.8 of this Chapter shall be permitted to be applied, with limiting maximum stress for shear taken as 55 percent of that given in Section 411.8.5. Permissible stress in shear-friction reinforcement shall be that given in Section 424.4.2

424.8.7 Special provisions for slabs and footings

424.8.7.1 Shear capacity of slabs and footings in the vicinity of concentrated loads of reactions is governed by the more severe of two conditions.

424.8.7.1.2 Two-way action for slab or footing, with a critical section perpendicular to plane of slab and located so that its perimeter is a minimum, but need not approach closer that d/2 to perimeter of the slab or footing shall be designed in accordance with Sections 424.8.7.2 and 424.8.7.3.

424.8.7.2 Design shears stress v shall be computed by

$$v = \frac{V}{b_o d} \qquad \text{Eq. 2-115}$$

Where v and b_o shall be taken at the critical section defined in Section 424.8.7.1.2

424.8.7.3 Design shear stress v shall not exceed v_c given by Eq. 2-166 unless shear reinforcement is provided

$$V_c = \frac{1}{12}\left(1+\frac{2}{\beta_c}\right)\sqrt{f'_c} \qquad \text{Eq. 2 -116}$$

But v_c shall not exceed (1/6) √f'c βc is the ratio of long side to short side of concentrated load or reaction area. When lightweight aggregate concrete is used, the modifications of Section 424.8.4.6 shall apply.

424.8.7.4 If shear reinforcement consisting of bars or wires is provided in accordance with Section 411.13.3 of this Chapter v_c shall not exceed $\frac{1}{12}$√f'c and v shall not exceed 1.25 √f'c.

424.8.7.5 If shear reinforcement consisting of steel |- or channel-shaped sections (shear heads) is provided in accordance with Section 411.13.2 of this Chapter , v on the critical section defined in Section 424.8.7.1.2 shall not exceed 0.3 √f'c and v on the critical section defined in Section Beam with total depth .not greater that 250mm Beam with total depth .not greater that 250mm Beam with total depth .not greater that 250mm 411.13.4.7 shall not exceed $\frac{1}{6}$√f'c. In Equations (411-39) and (411-40), design shear force V shall be multiplied by 2 and substituted V_u.

424.8.8 Special provisions for other members

For design of deep flexural members, brackets and corbels, and walls, the special provisions of Section 411 of this Chapter shall be used, with shear strengths provided by concrete and limiting maximum strengths for shear taken as 55 percent of the values given in section 411. In section 411.11.6 the design axial load shall be multiplied by 1.2 if compression and 2.0 if tension, and substituted for N_u.

424.8.9 Composite concrete flexural members

For design of composite concrete flexural members, permissible horizontal shear stress V_h shall not exceed 55 percent of the horizontal shear strengths given in Section 417.6.2 of this Chapter.

424.8.8 Special provisions for other members

For design of deep flexural members, brackets and corbels, and walls, the special provisions of Sections 411 of this Chapter shall be used, with shear strengths provided by concrete and limiting maximum strengths for shear taken as 55 percent of the values given in Section 4411. In Section 411.11.6 The design axial load shall be multiplied by 1.2 if compression and 2.0 if tension, and substituted for Nu.

424.8.9 Composite concrete flexural members

For design of composite concrete flexural members permissible horizontal shear stress V_n shall not exceed 55 percent of the horizontal shear strengths given in Section 417.6.2 of this Chapter.

STRUCTURAL DESIGN

BASIC CODE REQUIREMENTS

Types of Construction (Sect. 4.2)

Three basic types of construction and associated design assumptions are permissible under the respective conditions stated hereinafter, and each will govern in a specific manner the size of members and the types and strength of their connection.

Type 1, commonly designated as rigid-frame (continuous frame), assumes that the beam-to-column connection have sufficient rigidity to hold virtually unchanged the original angles between intersecting members.

Type 2, commonly design as simple framing (unrestrained, free-ended), assumes that insofar gravity loading is connected for shear only, and are free to rotate under gravity load.

Type 3, commonly designated as semi-rigid framing (partially restrained), assumes that the connections of beams and girder possess a dependable and known moment capacity intermediate in degree between the rigidity of Type 1 and the flexibility of Type 2.

The design of all connections shall be consistent with the assumptions as to type of construction called for on the design drawings.

Type 1 constructions unconditionally permitted under this Specification. Two different methods of design are recognized. Within the limitation laid down in Sect, 4.27, members of continuous portions of frames may be proportioned on the basic of their maximum predictable strength to resist the specific design loads multiplied by the prescribed load factors, Otherwise, Type 1 construction shall be designed, within the limitations of Sect. 4.5, to resist the stresses produced by the specified design loads, assuming moment distribution in accordance with the elastic theory.

Type 2 construction is permitted under this Specification, subject to the situation of the following paragraph, whenever applicable. In building designed as Type 2 construction (i.e., with beam-column connection other than wind connection assumes flexible under gravity loading) the wind moments may be distributed among selected joints of the frame, provided that:

1. The connections and connected members have adequate capacity to resist the wind moments.
2. The girders are adequate to carry the full gravity load as “simple beams”.
3. The connections have adequate inelastic rotation capacity to avoid overstresses of the fastener or welds under combined gravity and wind loading.

Type 3 (semi-rigid) construction will be permitted only upon evidence that the connections to be used are capable of furnishing, as a minimum, a predictable proportion of full end restraint. The proportioning of main members joined by such connection shall be predicated upon on greater degree of end restraint that this minimum.

Type 2 and 3 construction may necessitate some none plastic, but self-limiting, deformation of a structural steel part.

LOADS AND FORCES (SECT. 4.3)

Dead Load

The dead load to be assumed in design shall consist of the weight of steelwork and all material permanently fastened thereto or supported thereby.

Live Load

The live load shall be that stipulated by the applicable code under which the structure is being, designed or that dictated by the conditions involved.

Impact

For structures carrying live loads, which induce impact, the assumed live load shall be increased sufficiently to provide for same. If not otherwise specified, the increase shall be:

For support of elevators……………………………………….100%

For cab operated traveling crane support
girders and their connections…………………………………25%

For pendant operated traveling crane support
girders and their connections……………………..…………10%

For support of light machinery, shaft or motor
drive, not less than……………………………………………..20%

For supports of reciprocating machinery or power driven units, not less than……………………………………..50%

For hangers supporting floors and balconies…………………33%

Table 3 – 1 ASTM Structural Steel Grades for Rolled Products

Steel Type	ASTM Designation		F_y		F_x	
			ksi	**MPa**	**ksi**	**MPa**
Carbon	A36		32	221	58-80	400-551
			36	248	58-80	400-551
	A529		42	290	60-85	414-586
High-Strength Low-Alloy	A441		40	276	60	414
			42	290	63	434
			46	317	67	462
			50	345	70	483
	A572-Grade	42	42	290	60	414
		50	50	345	65	448
		60	60	414	75	517
		65	65	448	80	551
Corrosion Restraint High-Strength Low-Alloy	A242		42	290	63	434
			46	317	67	462
			50	345	70	483
	A588		42	290	63	434
			46	317	67	462
			50	345	70	483
Quench & Tempered Alloy	A514		90	620	100-130	689-896
			100	689	110-130	758-896

Table 3 – 2 Allowable Stresses for Structural Steel (Section 4.5)

Types of Stresses & Condition	NSCP Specification	Eq.
Tension:		
Except for pin-connected members		
1. On Gross Area	$F_t = 0.60\, F_y$	A
2. On Effective Net Area	$F_t = 0.50\, F_u$	B
For pin-connected members		
1. On Net Area	$F_t = 0.45\, F_y$	C
Shear:		
1. On effective cross-sectional area (except at reduced section, the effective area of rolled and fabricated shape may be taken as the overall depth times the web thickness)	$F_v = 0.40\, F_y$	D
2. At beam end connections where the top flange is coped, and in similar situations where failure might occur by shear along a plane through the fasteners, or by a combination of shear along a plane through the fasteners plus tension along a perpendicular plane (At reduced section)	$F_v = 0.30\, F_y$	E
Compression members/Columns:		
1. When $kL/r < C_c$ $C_c = \sqrt{\dfrac{2\pi^2 E}{F_y}}$ $FS = \dfrac{5}{8} + \dfrac{3(\frac{kL}{r})}{8C_C} - \dfrac{(\frac{kL}{r})^3}{8{C_c}^3}$	$F_a = \left[1 - \dfrac{\left(\frac{kL}{r}\right)^2}{2{C_c}^2}\right]\dfrac{F_y}{FS}$	F
2. When $kL/r > C_c$	$F_a = \left[\dfrac{12\pi^2 E}{23\left(\frac{kL}{r}\right)^2}\right]$	G

Types of Stresses & Condition	**NSCP Specification**	**Eq.**
3. On axially loaded bracings & secondary members where L/r >120	$F_a = \dfrac{F_a \text{ [by Eq. F or Eq. G]}}{1.6 - \dfrac{L}{200r}}$	H
4. On gross area of plate girder stiffeners	$F_a = 0.60\ F_y$	I
5. On web of rolled shapes a toe of fillet (cripping)	$F_a = 0.75\ F_y$	J
Bending on Strong Axis of I-shaped Members and Channels		
Compact Section		
Tension & Compression (provided the flanges are connected continuously to the web or webs and the laterally unsupported length of the compression flange L_b does not exceed the value of L_c, where L_c is the smaller value of $\dfrac{170}{\sqrt{F_y}}$ or $\dfrac{137{,}900}{\left(\dfrac{d}{A_f}\right)F_y}$	$F_b = 0.66\ F_y$	K
Non-compact Section		
For members meeting the above requirements for L_b except that their flanges are non-compact. i.e. $\dfrac{170}{\sqrt{F_y}} < \dfrac{b_f}{2t_f} < \dfrac{250}{\sqrt{F_y}}$	$F_b = F_y\left[0.79 - 0.00076\dfrac{b_f}{2t_f}\sqrt{F_y}\right]$	L
For non-compact section not included above with $L_b \le \dfrac{200b_f}{\sqrt{F_y}}$	$F_b = 0.60\ F_y$	M
Allowable bending stress in tension on compact or non-compact section with $L_b > L_c$	$F_b = 0.60\ F_y$	N

Types of Stresses & Condition	NSCP Specification	Eq.
Allowable ending stress in compression on compact or non-compact sections with $L_b > L_c$, F_b is the larger value of (Eq. O or Eq. P and Eq. Q):		
When: $\frac{L}{r_T} \geq \sqrt{\frac{703,270\ C_b}{F_y}}$ And $\frac{L}{r_T} \leq \sqrt{\frac{3,516,330C_b}{F_y}}$	$F_b = \left[\frac{2}{3} - \frac{F_y\left(\frac{l}{r_T}\right)^2}{10.55 \times 10^6 C_b}\right] F_y$ But $F_b \leq 0.60\ F_y$	O
When: $\frac{L}{r_T} > \sqrt{\frac{3,516,330C_b}{F_y}}$	$F_b = \frac{1,172,100\ C_b}{\left(\frac{l}{r_T}\right)^2} \leq 0.6F_y$	P
For any value of L/r_T Note: Eq. Q is applicable only to sections with compression flange that is solid and approximately rectangular in cross section and that has an area not less than the tension flange	$F_b = \frac{82,740\ C_b}{\left(\frac{ld}{A_f}\right)} \leq 0.6F_y$ For channels bent about their major axis, F_b in compression is determine from Eq. Q	Q
Bending on Weak Axis of I-shaped Members, Solid Bars, and Rectangular Plates		
Doubly symmetrical I- and H-shape members with compact flanges continuously connected to the web and bent about their weaker axis; solid round and square bars; and solid rectangular sections bent about their weaker axes	$F_b = 0.75F_y$	R
Members with non-compact section	$F_b = 0.60F_y$	S

Types of Stresses & Condition	NSCP Specification	Eq.
Bending on Weak Axis of Box Members, Rectangular Tubes and Circular Tubes		
Compact Section	$F_b = 0.66\,F_y$	T
Non-compact Section	$F_b = 0.66\,F_y$	U
Bearing		
On contact area of milled surfaces and ends of fitted bearing stiffeners; on projected area of pins reamed, drilled, or bored holes	$F_p = 0.90F_y$	V
On projected area of bolts and rivets in shear connections.	$F_p = 1.50F_u$	W

GROSS AND NET AREAS

Gross area

The gross area of a member at any point shall be determined by summing the product of the thickness and the gross width of each elements as measured normal to the axis of the member.

For angles, the gross width shall be the sum of the widths of the legs less the thickness.

Net Area

The net area A_n of a member is the sum of the products of the thickness and the net width of each element computed as follows:

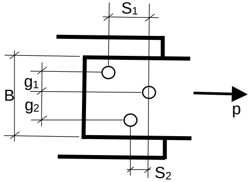
1. The width of bolt or rivet hole shall be taken as 1.6mm greater than the nominal dimension of the hole.

2. In the case of chains of holes extending across a part in any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain and adding, for each gage space in the chain, the quantity

$$s^2 / 4g \qquad \text{Eq. 3 – 1}$$

Where s = longitudinal center-to-center spacing (pitch) of any two consecutive holes, mm

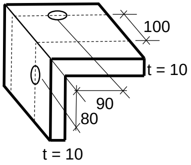
g = traverse center-to-center spacing (gage) of the same two holes, mm



$$A_n = \left[B - \sum \text{Holes} + \sum \frac{S^2}{4g}\right] t \qquad \text{Eq. 3-2}$$

$$A_e = A_n \leq 85\%\, A_g \qquad \text{Eq. 3-3}$$

3. For angels, the gross width shall be the sum of the widths of the legs less the thickness. The gage for holes in opposite legs shall be the sum of the gages from back of angles less the thickness



S = 100

g = 90 + 80 – 10 = 160

4. The critical net area A_n of the part is obtained from the chain which gives the least net width.
5. In determining the net area across plug or slot welds, the weld metal shall not be considered as a adding to the net area.

Effective Net Area

When the load is transmitted directly o each of the cross-sectional elements by connectors, the effective net area A_e is equal to the net area A_n.

When the load is transmitted by bolts or rivets through some but not all of the cross-sectional elements of the member the effective net area A_e shall be computed as:

$$\boxed{A_e = U\,A_n \qquad \text{Eq. 3 – 4}}$$

Where: A_n = net area of the member

U = a reduction coefficient

When the load is transmitted by welds through some but not all of the cross-sectional elements of the member, the effective net area A_e shall be computed as:

$$A_e = U\,A_g \qquad \text{Eq. 3 – 4}$$

Where: A_g = gross area of the member

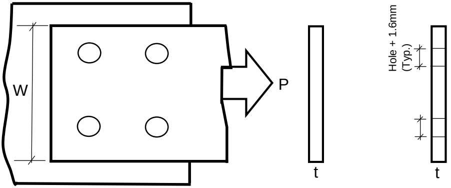
Unless a large coefficient can be justified by test or other recognized criteria, the following values of C_t shall be used in computations:

1. W, M or S shapes with flange width not less than 2/3 the depth, and structural tees cut from these shapes, provided the connection is to the flanges and has no fewer than 3 fasteners per line I the direction of stress, U = 0.90
2. W, M or S shapes not meeting the conditions of subs paragraph1, structural tees cut from these shapes, including has not than fasteners per line in the direction of stress, U = 0.85.
3. All members whose connections have only 2 fasteners per line in the direction of stress, U = 0.755.

Riveted and bolted splice and gusset plates and other connection fitting subject to tensile force shall be designed in accordance with the provisions of Sect. 4.5.1.1, where the effective net area shall be taken as the actual net area, except that, for the purpose of design calculations, it shall not be taken a greater than 85 percent of the gross area.

DESIGN OR ANALYSIS OF RIVETED OR BOLTED AXIALLY LOADED TENSION MEMBERS (CONNECTIONS)

The following stresses must be verified in the design or analysis of axially loaded tension connections:



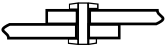
Gross Area A_g Net Area $A_n \le 0.85\,A_g$

Gross Area, $A_g = W \times t$
Net Area, $A_n = [W - \sum(\text{holes}+1.6)] \times t \le 85\%\ A_g$

1. Tension on Gross Area:

Actual Stress $f_t = P/A_g$

Allowable Stress, $F_t = 0.60\ F_y$

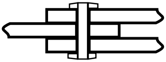


Single Shear

2. Tension on Net Area:

Actual Stress, $f_t = P/A_n$

Allowable Stress, $F_t = 0.50\ F_u$



Double Shear

3. Shear in Bolts

Actual Stress, $f_t = P/A_v$

$A_v = A_{bolt} \times n$ (for single shear)

$A_v = 2A_{bolt} \times n$ (for double shear)

n = number of bolts

Allowable shearing stress, F_v

depends on the type and material of bolts

See Table 3 – 9

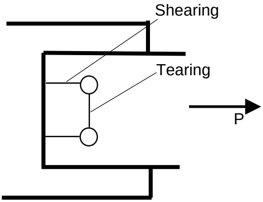
4. Bearing on the projected area between the bolt and the plate:

Actual Stress, $f_p = P/A_p$

$A_p = \sum$(Bolt diameter x plate thickness)

Allowable stress $F_p = 1.50\ F_u$

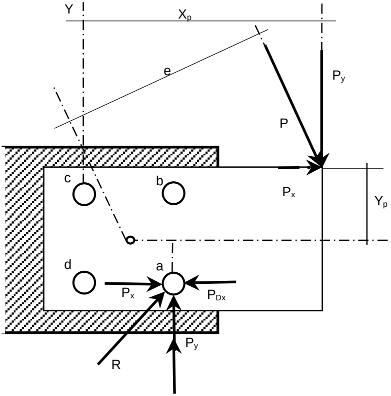
5. Combined shearing and tearing:



Allowable shearing stress, $F_v = 0.30\ F_u$

Allowable tearing stress, $F_t = 0.50\ F_u$

ECCENTRICALLY LOADED BOLTED/RIVETED CONNECTION



$$\text{Direct Load } P_{Dy} = \frac{P_y}{n} \ \& \ P_{Dx} = \frac{P_x}{n} \qquad \text{Eq. } 3-6$$

$$\text{Moment } T = Pe = P_y(X_p) + P_x(Y_p) \qquad \text{Eq. } 3-7$$

$$P_x = \frac{Ty}{\sum(x^2 + y^2)} \qquad \text{Eq. } 3-8$$

$$P_y = \frac{Tx}{\sum(x^2 + y^2)} \qquad \text{Eq. } 3-9$$

$$\text{Total Load, } R = \sqrt{(P_x + P_{Dx})^2 + (P_y + P_{Dy})^2} \qquad \text{Eq. } 3-10$$

Where n = number of rivets

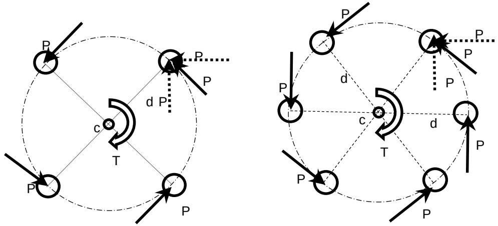
x = x-coordinate of the rivet

y = y-coordinate of the rivet

P_x & P_y = load due to moment alone

P_{Dx} & P_{Dy} = load due to axial force alone (direct load)

If the rivets are equidistant from the centroid of the rivet group such as those shown below:



(P X d) n =T

Load due to moment alone, $P = \dfrac{T}{n \times d}$	Eq. 3 - 11

WELDED CONNECTION

GROOVE WELDS

Effective Area of Groove Welds

The effective area of groove welds shall be based on the following:

1. The effective area of a groove welds shall be considered as the effective length of the weld time the effective throat thickness.
2. The effective length a groove weld shall be the width of the part joined.
3. The effective throat thickness of a complete penetration groove weld shall be the thickness of the thinner part joined.
4. The effective throat thickness of a partial-penetration groove weld shall be as shown in Table 3 – 3.
5. The effective throat thickness of a flare groove weld when flush to the surface of a bar of 90˚ bend in a formed section shall be as shown in Table 3 – 4. Random sections of production welds for each welding procedure, or such test sections as may be required by design documents, shall be used to verify that effective throat is consistently obtained.
6. Larger effective throat thicknesses than those in Table 3 – 3 are permitted, provided the fabricator can establish by qualification that he can consistently provide such larger effective throat thicknesses. Qualification shall consist of sectioning the weld normal to its axis, at mid – length and terminal ends. Such sectioning shall be made on a number of combinations of materials sizes representative of the range to be used in the fabrication or as required by the designer.

Table 3 – 3: Effective Throat Thickness of Partial-Penetration Groove Welds

Welding Process	Welding Position	Included Angle at Root of Groove	Effective Root Thickness

Shielded metal arc Submerged arc	All	J or U Joint	Depth of chamfer
Gas Metal Arc		Bevel or V joint ≥ 60˚	
Flux-cored arc		Bevel or V joint < 60˚ but ≥ 45˚	Depth of chamfer minus 3mm

Table 3 – 4: Effective Throat Thickness of Flare Groove Welds

Type of Weld	**Radius (R) of Bar or Bend**	**Effective Throat Thickness**
Flare Bevel Groove	All	(5/16)R
Flare V-Groove	All	(1/2)R[t]
[t] Use (3/8)R for Gas Metal Arc Welding (Except short circuiting transfer process) when R ≥ 12mm		

Table 3 – 5: Minimum Effective Throat Thickness of Partial – Penetration Groove Welds

Material Thickness of Thicker Part Joined	**Minimum Effective Throat Thickness**
To 6 mm inclusive	3 mm
Over 6 mm to 12 mm	5 mm
Over 12 mm to 20 mm	6 mm
Over 20 mm to 38 mm	8 mm
Over 38 mm to 57 mm	10 mm
Over 57 mm to 150 mm	12 mm
Over 150 mm	16 mm

Limitations of Groove Weld

The minimum effective throat thickness of a partial-penetration groove weld shall be as shown in Table 3 – 5. Minimum effective throat thickness is determined by the thicker of two parts joined, except that the weld size need not exceed the thickness of the thinnest part joined. For this exception, particular care shall be taken to provide sufficient preheat for soundness of the weld.

FILLET WELDS

Effective Area

The effective area of groove welds shall be based on the following:

1. The effective area of fillet welds shall be taken as the effective length times the effective throat thickness.
2. The effective length of fillet welds, except fillet welds in holes and slots, shall be the overall length of full-size fillets, including returns.
3. The effective throat thickness of a fillet weld shall be the shortest distance from the root of the joint to the face of the diagrammatic weld, except that for fillet welds made by the submerged arc process, the effective throat thickness shall be taken equal to the leg size for 10mm and smaller fillet welds, and equal to the theoretical throat plus 3 mm for fillet welds larger than 10 mm.
4. For fillet welds in holes and slots, the effective length shall be the length of the centerline of the weld along the center of the lane through the throat. In the case of overlapping fillets, the effective area shall not exceed the nominal cross-sectional area of the hole slot in the plane of the faying surface.

Limitation of Fillet Welds

1. The minimum size of fillet welds shall be as shown in Table 3 – 6. Minimum weld size is dependent upon the thicker of the two parts joined, except that the weld size

need not exceed the thickness of the thinner part. For this exception particular case shall be taken to provide sufficient preheat for soundness of the weld. Weld sizes larger than the thinner art joined are permitted if required by calculated strength. In the as-welded condition, the distance between the edge of the base metal and the toe of the weld may be less than 1.6 mm provided the weld size is clearly verifiable.

Table 3 – 6: Minimum Size of Fillet Welds

Material Thickness of Thicker Part Joined	Minimum Size of Fillet Weld
To 6 mm inclusive	3 mm
Over 6 mm to 12 mm	5 mm
Over 12 mm to 20 mm	6 mm
Over 20 mm	8 mm

2. The maximum size of fillet welds that is permitted along edges of connected parts shall be:
   a. Material less than 6 mm thick, not greater than the thickness of the material.
   b. Material 6 mm or more in thickness, not greater than the thickness of the material minus 1.6 mm, unless the weld is especially designated on the drawings to be built out to obtain full-throat thickness.
3. The minimum effective length of fillet welds designated on the basis of strength shall be not less than 4 times the nominal size, or else the size of the weld shall be considered not to exceed ¼ of its effective length. If longitudinal fillet welds are used alone in end connections of flat bar tension members, the length of each fillet weld shall be not less than the perpendicular distance between

them the traverse spacing of longitudinal fillet welds used in end connections of tension members shall not exceed 200 mm, unless the member is designed on the basis of effective net area.

4. Intermittent filled welds are permitted to transfer calculated stress across a joint or faying surface when the strength required is less than that developed by a continuous filled weld of the smallest permitted size, and to join components of built-up members. The effective length of any segment of intermittent fillet welding shall be not less than 4 times the weld size, with a minimum of 38 mm.
5. In lap joints, the minimum lap shall be 5 times the thickness of the thinner part joined, but not less than 25 mm. lap joints joining plates or bars subjected to axial stress shall be fillet welded along the end of both lapped parts, except where the deflection of the lapped part is sufficiently restrained to prevent opening of the joint under maximum loading.
6. Fillet welds in holes or lots are permitted to transmit shear in lap joints or to prevent the buckling or separation of lapped parts and to join components of built-up member such fillet welds may overlap, subject to the provision of the Code. The fillet welds in holes or slots are not to be considered plug or slot welds.
7. Slide or end fillet welds terminating at ends or sides, respectively, of parts members shall, wherever practicable, be returned continuously around the corners for a distance not less than 2 times the normal size of the weld. This provision shall apply to side and top fillet welds connecting brackets, beam seats ad similar connections,

on the plane about which bending moments are computed. For framing angles and simple end-plate connections which depend upon flexibility of the outstanding legs for connection flexibility, end returns shall not exceed four time the nominal size of the weld. Fillet welds that occur on opposite side of a common to both welds. End returns shall be indicated on the design and details drawings.

PLUG AND SLOT WELDS

Effective Area

The effective shearing area of plug and slots welds shall be considered as the nominal cross-sectional area of the hole or slot in the plane of the faying surface.

Limitations of Plug and Slot Welds

1. Plug or slot welds are permitted to transmit shear in lap joints or to prevent buckling of lapped arts and to join component parts of built-up members.
2. The diameter of the hole for a plug weld shall not be less than the thickness of the art containing it plus 8 mm, rounded to the next larger odd 1.6 mm, nor greater than the minimum diameter plus 3 mm or 2 ¼ times the thickness of the weld.
3. The minimum center-to-center spacing of plug welds shall be four times the diameter of the hole.
4. The minimum spacing of lines of slot welds in a direction transverse to their length shall be 4 times the width of the slot. The minimum center-to-center spacing in a

longitudinal direction on any line shall be 2 times the length of the slot.

5. The length of slot for a slot weld shall not exceed 10 times the thickness of the weld. The width of the slot shall be not less than the thickness of the part containing it plus 8 mm, nor shall it be larger than 2v4 times the thickness of the weld. The ends of the slot times shall be semi-circular or shall have the corners rounded to a radius not less than the thickness of the part containing it, except those ends which extend to the edge of the part.
6. The thickness of plug or slots welds in material 16 mm or less in thickness shall be equal to the thickness of the material. In material over 16 mm thick, the thickness of the weld shall be at least ½ the thickness of the material but not less than 16 mm.

COMBINATION OF WELDS

If two or more of the general types of weld (groove, fillet, plug, slot) are combined in a single joint, the effective capacity of each shall be separately computed with reference to the axis of the group in order to determined allowable capacity of the combination.

Mixed Weld Metal

When notch-toughness is specified, the process consumables for all weld metal, tack welds, root pass and subsequent passes, deposited in a joint shall be compatible to assure notch-tough composite weld metal.

Table 3 – 7: Allowable Stresses on Welds

<table>
<tr><th>Type of Weld and Stress</th><th>Allowable Stress</th><th>Required Weld Strength Leve [b, c]</th></tr>
<tr><td colspan="3">Complete-Penetration Groove Welds</td></tr>
<tr><td>Tension normal to effective area</td><td>Same as base metal</td><td>“Matching” weld metal must be used</td></tr>
<tr><td>Compression normal to effective area</td><td>Same as base metal</td><td rowspan="3">Weld metal with a strength level equal to or less than “matchmaking” weld metal may be used.</td></tr>
<tr><td>Tension and compression parallel to axis of weld</td><td>Same as base metal</td></tr>
<tr><td>Shear on effective area</td><td>0.3 x nominal tensile strength of weld metal (MPa) except shear stress on base metal shall not exceed 0.40 x yield stress of base metal</td></tr>
<tr><td colspan="3">Partial-Penetration Groove Welds[d]</td></tr>
<tr><td>Compression normal to effective area</td><td>Same as base material</td><td rowspan="4">Weld metal with a strength level equal to or less than “matchmaking” weld metal may be used</td></tr>
<tr><td>Tension and compression parallel to axis of weld</td><td>Same as base material</td></tr>
<tr><td>Shear parallel to axis of weld</td><td>0.3 x nominal tensile strength of weld metal (MPa) except shear stress on base metal shall not exceed 0.40 x yield stress of base metal</td></tr>
<tr><td>Tension normal to effective area</td><td>0.3 x nominal tensile strength of weld metal (MPa) except shear stress on base metal shall not exceed 0.60 x yield stress of base metal</td></tr>
<tr><td colspan="3">Fillet Welds</td></tr>
<tr><td>Shear on effective area</td><td>0.3 x nominal tensile strength of weld metal (MPa) except shear stress on base metal shall not exceed 0.40 x yield stress of base metal</td><td rowspan="2">Weld metal with a strength level equal to or less than “matchmaking” weld metal may be used</td></tr>
<tr><td>Tension or compression parallel to axis of weld</td><td>Same as base material</td></tr>
<tr><td colspan="3">Plug and Slot Welds</td></tr>
<tr><td>Shear parallel to faying</td><td>0.3 x nominal tensile strength of weld metal (MPa) except shear stress on base metal shall not exceed 0.40 x yield stress of base metal</td><td>Weld metal with a strength level equal to or less than “matchmaking” weld metal may be used</td></tr>
</table>

a. For definition of effective area, see Section 4.14.6
b. For “matching” weld metal, see Table 4.1.1, AWS D1.1-77
c. Weld metal one strength level stronger than the “Matching” weld metal will be permitted.
d. See Sect. 4.10.8 for a limitation on use of partial-penetration groove welded joints.
e. Fillet welds and partial-penetration groove welds joining the component elements of built-up members, such as flange-to-web connections, may be designed without regard to the tensile or compressive stress in these elements parallel to the axis of the welds.

The permissible unit stresses for fillet welds made with E 60 XX -, E 70 XX -, and E 80 XX – type electrodes on A 36 the fact that the stress in a fillet weld is considered as shear on the throat, regardless of the direction of the applied load. Neither plug nor slot welds shall be assigned any values in resistance other that shear.

Table 3 – 8: Allowable Working Strength of Fillet Welds

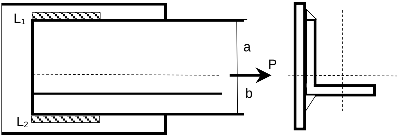
Size of Weld		**Allowable Load (kN/mm)**		
		E 60 xx Electrode F_u = 60 ksi F_u = 414 MPa F_v = 0.3 F_u F_v = 124 MPa	**E 70 xx Electrode F_u = 70 ksi F_u = 482 MPa F_v = 0.3 F_u F_v = 145 MPa**	**E 80 xx Electrode F_u = 80 ksi F_u = 551 MPa F_v = 0.3 F_u F_v = 165 MPa**
in	**mm**			
3/16	4.76	0.417	0.488	0.555
1/4	6.35	0.557	0.651	0.741
5/16	7.94	0.696	0.814	0.926
3/8	9.52	0.835	0.976	1.111
1/2	12.7	1.113	1.302	1.482
5/8	15.9	1.394	1.630	1.855
3/4	19.2	1.683	1.968	2.240

Fillet Weld

Throat = 0.717 t	Eq. 3 – 12
Capacity, $P = F_v\,(0.707\,t\,L)$	Eq. 3 – 13

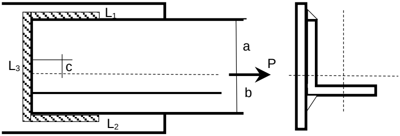
BALANCING WELD

Angular Section Fillet Welded on a Gusset Plate



$P = 0.707\,t\,L\,F_v$	Eq. 3 – 14
$L = L_1 + L_2$	Eq. 3 – 15
$L_1 \times a = L_2 \times b$	Eq. 3 – 16

Angular Section Fillet Welded on a Gusset Plate (With Transverse Fillet Weld)



$$P = 0.707\ t\ L\ F_v \qquad \text{Eq. 3 – 17}$$
$$L = L_1 + L_2 + L_3 \qquad \text{Eq. 3 – 18}$$
$$L_1 \times a = L_2 \times b = L_3 \times c \qquad \text{Eq. 3 – 19}$$

ECCENTRICALLY LOADED WELDED CONNECTION

$$\text{Direct Load, } P_D = \frac{F}{\sum L} \qquad \text{Eq. 3 – 20}$$
$$\text{Moment, } T = F \times e \qquad \text{Eq. 3 – 21}$$
$$P_x = \frac{Ty}{J} \qquad \text{Eq. 3 – 22}$$
$$P_y = \frac{Tx}{J} \qquad \text{Eq. 3 – 23}$$
$$J = \sum L \left[\frac{L^2}{12} + X_G{}^2 + Y_G{}^2\right] \qquad \text{Eq. 3 – 24}$$
$$\text{Total load, } R = \sqrt{(P_x)^2 + (P_x + P_D)^2} \qquad \text{Eq. 3 – 24}$$

Where P_D= direct load in $\frac{N}{mm}$

P_x & P_y= load due to moment in $\frac{N}{mm}$

L=length of each weld, mm

e = eccentricity, mm

Table 3 – 9: Allowable Stresses on Fasteners, MPa

Description of Fasteners	Allowable Tension[g] F_t	Allowable Shear[g] (F_v)			
		Friction-type Connections[e,i]			Bearing-type connection
		Standard Size Holes	Oversized and Short-slotted Holes	Long-slotted Holes	
A 502 Grade 1, hot-driven rivets	160[a]				120[f]

A 502, Grade 2 & 3, hot-driven rivets	200[a]				150[f]
A 307 Bolts	140[a]				70
Threaded parts meeting the requirements of Sects. 4.4.1 & 4.4.4 and A449 bolts meeting the requirements of Sect. 4.4.4, when threads are not excluded from shear planes	$0.33F_u$ [a,c,h]				$0.17F_u$[h]
Threaded parts meeting the requirements of Sects. 4.4.1 & 4.4.4 and A449 bolts meeting the requirements of Sect. 4.4.4, when threads are excluded from shear planes	$0.33F_u$ [a,h]				$0.22F_u$[h]
A 325 bolts, when treads are not excluded from shear planes	300[d]	120	100	90	145[f]
A 325 bolts, when treads are excluded from shear planes	300[d]	120	100	90	210[f]
A490 bolts, when threads are not excluded from shear planes	370[d]	150	130	110	190[f]
A490 bolts, when threads are excluded from shear planes	370[d]	150	130	110	280[f]

a. Static loading only
b. Thread permitted in shear lanes
c. The tensile capacity of the threaded portion of an upset rod, based upon the cross-sectional area of its major thread diameter, A_b, shall be larger than the nominal body area of the rod before upsetting times 0.60 F_y.
d. For A325 and A490 bolts subject to tensile fatigue loading, see Appendix B, Sect B3
e. When specified by the designer, the allowable shear stress, F_v, for friction-type connections having a special faying surface conditions may be increased to the applicable value given in Appendix E.
f. When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 1270 mm, tabulated values shall be reduced by 20%.
g. See Sect 4.5.6
h. See Appendix C, Table 2, for values specific ASTM steel specifications
i. For limitations on use of oversized and slotted holes, see Sect. 4.23.4.

AXIALLY LOADED COLUMNS & OTHER COMPRESSION MEMBERS

Euler's Stress

For Hinged-Ended Columns:

Euler critical load, $P = \frac{\pi^2 EI}{L^2}$ Eq. 3 – 26

Euler critical stress, $F_a = \frac{\pi^2 E}{(L/r)^2}$ Eq. 3 – 27

For Fixed-Ended Columns:

Euler critical load, $P = \frac{4\pi^2 EI}{L^2}$ Eq. 3 – 28

Euler critical stress, $F_a = \frac{4\pi^2 E}{(L/r)^2}$ Eq. 3 – 29

Where L = unbraced length

L/r = maximum slenderness ratio

<u>NSCP/AISC Specifications</u>

$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$ Eq. 3 – 30

KL/r = Maximum effective slenderness ratio

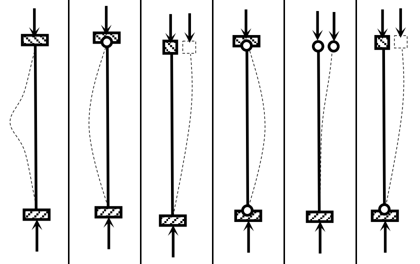
K = effective length factor (See Table 3-10 in Page 255)

K = 1 for column hinged at both ends

K = 0.5 Fixed-Fixed

K = 0.7 Hinge-Fixed

Table 3 -10: Effective Length Factors

Buckled shape of column is shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.20	1.00	2.10	2.00
End conditions code		Rotation fixed and translation fixed				
		Rotation free and translation fixed				
		Rotation fixed and translation free				
		Rotation free and translation free				

When KL/r < C_s (short column)

$$F_a = \left[1 - \frac{(KL/r)^2}{2C_c^{\,2}}\right]\frac{F_y}{FS} \qquad \text{Eq. 3 – 31}$$

$$FS = \frac{5}{3} + \frac{3\,(K\,L/\,r)}{8\,C_c} - \frac{(K\,L\,/\,r)^3}{8\,C_c^{\,3}} \qquad \text{Eq. 3 – 32}$$

When KL/r > C_c (long column)

$$F_s = \frac{12\,\pi^2 E}{23\,(KL/r)^2} \qquad \text{Eq. 3 – 33}$$

Allowable Stress on Axially Loaded Bracings and Secondary Members where L/r > 120

$$F_a = \frac{1}{1.6 - \frac{L}{200\,r}}\ F_a \text{ [by Eq. 3-31 or Eq. 3-33]} \qquad \text{Eq. 3 – 34}$$

<u>BEAMS AND OTHER FLEXURAL MEMBERS</u>

These formulas apply to singly or doubly symmetric beams including hybrid beams and girders loaded in the plane of symmetry. It also applies to channels loaded in a plane passing through the shear center parallel to the web or restrained against twisting at load points and points of support

<u>ALLOWABLE STRESS ON STRONG AXIS BENDING OF I – Shaped Members and Channels</u>

Members with Compact Section
(See Table 3 – 11 for limiting width – thickness ratio)

Criteria for Compact Section of I – Shaped members and Tees:

Width-thickness ratio, $\frac{b_f}{2t_f} \leq \frac{170}{\sqrt{F_y}}$ Eq. 3 – 35

Depth-thickness ratio, $\frac{d}{t_w} \leq \frac{1680}{\sqrt{F_y}}$ Eq. 3 – 36

For the member with compact section (excluding hybrid beams and members with yield points greater than 448 MPa), the allowable bending stress in both tension and compression is:

F_b=0.66 F_y Eq. 3 – 37

provided the flanges are connected continuously to the web or webs and the laterally unsupported length of compression flange L_b does not exceed L_c, where L_c is the smaller value of Eq. 3 – 38 and Eq. 3 – 39.

$\frac{200\,b_f}{\sqrt{F_y}}$ Eq. 3 – 38

$\frac{137{,}900}{(d/A_f)\,F_y}$ Eq. 3 – 39

Members with non – Compact Section

(See Table 3 – 11 for limiting width – thickness ratios)

For members with $L_b \leq L_c$ except that their flanges are non – compact (excluding built-up members and members with yield points greater than 448 MPa), the allowable bending stress in both tension and compression is

$$F_b = F_y \left[0.79 - 0.000762 \frac{b_f}{2t_f} \sqrt{F_y}\right] \qquad \text{Eq. 3 – 40}$$

For members with non – compact section (not included in the above) and $L_b \leq \frac{200 b_f}{\sqrt{F_y}}$, the allowable bending stress in both tension and compression is:

$$F_b = 0.60\, F_y \qquad \text{Eq. 3 – 41}$$

Members with Compact or Non – Compact Section With $L_b > L_c$

Allowable bending stress in tension:

$$F_b = 0.60\, F_y \qquad \text{Eq. 3 – 42}$$

Allowable bending stress in compression:
The allowable bending stress in compression is determined as the larger value of (Eq. 3-43 or Eq. 3-44 and Eq. 3-45, except that Eq. 3-45 is applicable only to section with compression flange that is solid and approximately rectangular in cross-section and that has an area not less than tension flange. For channels, the allowable compressive stress is determined from Eq. 3-45.

When $\sqrt{\frac{703{,}270\, C_b}{F_y}} \leq \frac{L}{r_T} \leq \sqrt{\frac{3{,}516{,}330\, C_b}{F_y}}$

$$F_b = \left[\frac{2}{3} - \frac{F_y (I/r_T)^2}{10.55 \times 10^6\, C_b}\right] F_y \leq 0.60\, F_y \qquad \text{Eq. 3 – 43}$$

Use the larger value of Eq. 3-45 and Eq. 3-43, but shall be less than 0.60 F_y.

When $\frac{L}{r_T} > \sqrt{\frac{3{,}516{,}330\ C_b}{F_y}}$

$$F_b = \frac{1{,}172{,}100\, C_b}{(I/r_T)^2} \leq 0.60\, F_y \qquad \text{Eq. 3 – 44}$$

Use the larger value of Eq. 3-45 and Eq. 3-43, but shall be less than 0.60 F_y.

For any value of L/r_T:

$$F_b = \frac{82{,}740\, C_b}{(Id/A_f)} \leq 0.60\, F_y \qquad \text{Eq. 3 – 45}$$

Where:

b_f = flange width, mm
t_f = flange thickness, mm
d = depth, mm
t_w = web thickness, mm
A_f = area of compression flange = $b_f\, t_f$ (mm^2)
I = distance between cross-sections braced against twist and lateral displacements of the compression flange mm

r_T = radius of gyration of the section comprising the compression flange plus 1/3 of the compression web area taken about an axis in the plane of the web mm

$C_b = 1.75 + 1.05\,(M_1/M_2) + 0.3\,(M_1/M_2)^2 \leq 2.3$

M_1 = smaller end moment; M_2 = larger end moment

M_1/M_2 = (+) for reversed curvature

M_1/M_2 = (-) for single curvature

C_b = unity (1) if the moment within the unbraced length is larger than M_1 or M_2. For example the simply supported beam.

Allowable Stress on Weak Axis Bending of I – Shaped Members, Solid Bars and Rectangular Plates

See Equations R & S in Table 3 – 2

SHEARING STRESS IN BEAMS

On the Cross-Sectional Area Effective in Resisting Shear:

When $\frac{h}{t_w} \leq \frac{998}{\sqrt{F_y}}$, the allowable shear stress on the overall depth time the web thickness (d t_w) is:

$$F_v = 0.40\,F_y \qquad \text{Eq. 3 – 46}$$

$$\text{or } \frac{V}{dt_w} \leq 0.40\,F_y \qquad \text{Eq. 3 – 47}$$

When $\frac{h}{t_w} \leq \frac{998}{\sqrt{F_y}}$, the allowable shear stress on the clear distance between flanges times the web thickness, h t_w is:

$$F_v = \frac{F_y}{2.89} C_v \leq 0.40 F_y \qquad \text{Eq. } 3 - 48$$

$$\text{or } 0.40 F_y \geq \frac{V}{h\, t_w} \leq \frac{F_y}{2.89} C_v \qquad \text{Eq. } 3 - 49$$

Where:

$C_v = \frac{310{,}264\, k_v}{F_y\,(h/t_w)^2}$ when C_v is less than 0.80

$C_v = \frac{500}{h/t_w}\sqrt{\frac{k_v}{F_y}}$ when C_v is more than 0.80

$k_v = 4.00 + \frac{5.34}{(a/h)^2}$ when a/h is less than 1.0

$k_v = 5.34 + \frac{4.00}{(a/h)^2}$ when a/h is more than 1.0

t_w = thickness of web, mm

a = clear distance between transverse stiffeners, mm

h = clear distance between flanges at the section under investigation, mm

$h = d - 2t_f$

d = overall depth of the beam, mm

COMBINED STRESSES

This section pertains to doubly and singly symmetrical members only. Use Eq. 3-31 or Eq. 3-31(as applicable) for determination of F_a and Eq. 3-37 through Eq. 3-45 (as applicable) for determination of F_{bx} and F_{by}.

Axial Compression and Bending

Members subjected to both axial compression and bending shall be proportioned to satisfy the following requirements

$$\frac{f_a}{F_a} + \frac{C_{mx}\, f_{mx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my}\, f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right)} \leq 1 \qquad \text{Eq. 3 – 50}$$

$$\frac{f_a}{0.60 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1 \qquad \text{Eq. 3 – 51}$$

When $f_a/F_a \leq 0.15$, Eq. 3-52 is permitted in lieu of Eq. 3-50 and Eq. 3-51.

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1 \qquad \text{Eq. 3 – 52}$$

In Eq. 3 – 50 through Eq. 3 – 52, the subscripts x and y, combined with subscripts b, m and e indicate the axis of bending about which a particular stress or design properly applies.

Where:

F_a = allowable axial compressive stress if axial force alone existed MPa

F_b = allowable compressive bending stress if bending moment alone existed

$F'_e = \frac{12\,\pi^2 E}{23\,(KI_b/r_b)^2}$ = Euler stress divided by a factor of safety, MPa (In the expression for F'_e I_b is the actual unbraced length in the plane of bending and r_b is the corresponding radius of gyration. K is the effective length factor in the plane of bending). As in the case of F_a, F_b and 0.60 F_y, F'_e may be increased 1/3.

f_a = computed axial stress, MPa

f_b = computed compressive bending stress at the point under consideration, MPa

C_m = a coefficient whose values is as follows:

1. For compression members in frames subject to joint translation (sidesway), $C_m = 0.85$
2. For rotationally restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending.

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \text{ but not less than 0.4}$$

   When M_1/M_2 is the ratio of the smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.
3. For compression members in frames braced against joint translation in the plane of loading and subjected to transverse loading between their supports, the value of C_m may be determined by rational analysis. However, in lieu of such analysis, the following values may be used:
   a. For members whose ends are restrained
      $C_m = 0.85$
   b. For members whose ends are unrestrained
      $C_m = 1.0$

Axial Tension and Bending

Members subjected to both axial tension and bending shall be proportioned at all points along their length to satisfy the requirements of formula:

$$\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1 \qquad \text{Eq. 3 – 53}$$

Where f_b is the computed bending tensile stress, f_a is the computed axial tensile stress, F_b is the allowable bending stress, and F_t is the governing allowable tensile stress.

COMPOSITE BEAMS

This section applies to steel beams supporting a reinforced concrete slab so interconnected that the beams and the slab so interconnected that the beams and the slab act together to resist bending. This also includes simple and continuous composite beams, constructed with or without temporary shores.

Composite beams may be:

1. Totally encased beams which depend upon natural bond for interaction with the concrete.
2. Beams with shear connectors (mechanical anchorage to the slab) with steel member not necessarily encased.

Encased Beams

A beam totally encased in concrete cast integrally with the slab may be assumed to be connected to the concrete by natural bond, without additional anchorage, provided that:

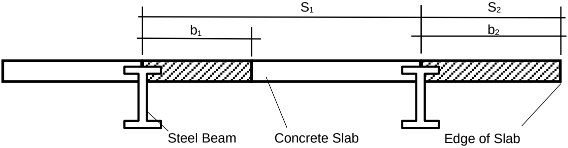
1. Concrete cover over beam sides and soffit is at least 50 mm
2. The top of the beam is at least 38 mm below the top and 50 mm above the bottom of the slab.
3. Concrete, encasement contains adequate mesh or other reinforcing steel throughout the whole depth and across the soffit of the beam to prevent spalling of the concrete.

Beams with Shear Connectors

Shear connectors must be provided for composite section if the steel member is not totally encased in concrete. The effective width of concrete flange on each side of the beam centerline shall not exceed:

1. One-eight of the beam span, center-to-center of supports:
2. One-half the distance to the centerline of the adjacent beam, or
3. The distance from the beam centerline to the edge of the slab.

   b = effective width of slab on one side of the beam



From the figure shown:

b_1, is the smallest of:

1. L / 8, where L is the beam span
2. $S_1 / 2$

b_2 shall not exceed S_2

Shear Connectors

Except in the case of encase beams, the entire horizontal shear at the junction of the steel beam and concrete slab shall be assumed to be transferred by shear connectors welded to the top flange of the beam and embedded in the concrete. For full composite action with concrete subjected to flexural compression, the total horizontal shear to be resisted between the point of maximum positive moment and points of zero moment shall be taken as the smaller value using Eq. 3 – 54 and Eq. 3 – 55.

$$V_h = 0.85 f'_c \; A_c / 2 \qquad \text{Eq. 3 – 55}$$

$$\text{and } V_h = A_s \; F_y/2$$

Where; f'_c = specified compression strength of concrete, MPa

A_c = the actual area of effective concrete flange, mm^2

A_s = the area of steel beam, mm^2

F_y = the specified yield strength of steel beam

Table 3 – 11: Limiting Width-Thickness Ration for Compression Members

Steel sections are classified as compact, non-compact, and slender element sections. For a section to qualify as compact, its flanges must be continuously connected to the web or webs and the width-thickness ratios of its compression elements must not exceed the limits values given in this table.

For unstiffened elements, which are supported along one edge only, parallel to the direction of compression force, the width shall be taken as follows:

1. For flanges of I-Shaped members and tees, b is ½ the full nominal width.
2. For legs of angles and flanges of channels and zees, b is the full nominal dimension.
3. For plates, b is the distance from the free edge to the first row of fasteners or line of welds.
4. For stems of tees, d is the full nominal depth.

For stiffened elements, i.e., supported along two edges parallel to the direction of the compression force the width shall be taken as follows:

1. For webs of rolled, built-up or formed sections, h is the clear distance between flanges.
2. For webs of rolled, built-up or formed sections, d is the full nominal depth.
3. For flanges or diaphragm plates in built-up section, b is the distance between adjacent lines of fasteners or lines of welds.
4. For flanges of rectangular hallow structural sections, b is the clear distance between webs less the inside corner radius of each side. If the corner radius is not known, the flat width may be taken as the total section width minus three times the thickness.

For tapered flanges of rolled sections, the thickness is the nominal value halfway between the free edge and the corresponding face of the web.

Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios	
		Compact	Non – Compact
Flanges of I-Shaped rolled beams and channels in flexure	b/t	$170/\sqrt{F_y}$	$250/\sqrt{F_y}$
Flanges of I-Shaped of welded beams in flexure	b/t	$170/\sqrt{F_y}$	$170/\sqrt{F_y/k_c^e}$
Outstanding legs if pairs of angels of continuous contact; angels or plates projecting from rolled beams or columns; stiffeners on plate girders	b/t	NA	$250/\sqrt{F_y}$
Angels or plates projecting from girders, built-up columns or other compression members; compression flanges of plate girders	b/t	NA	$170/\sqrt{F_y/k_c}$

Unsupported width of cover plates perforated with a succession of access holes	b/t	NA	$832/\sqrt{F_y}$
All other uniformly compressed stiffened elements, i.e., supported along two edges.	b/t h/t_w	NA	$664/\sqrt{F_y}$
Webs in flexural compression	d/t	$1680/\sqrt{F_y}$	--
	h/t_w	--	$1995/\sqrt{F_y}$
Webs in combined flexural and axial compression	d/t_w	For $f_a/F_y \leq 0.16$ $\frac{1680}{\sqrt{F_y}}\left(1\text{-}3.74\ \frac{f_a}{F_y}\right)$	--
		For $f_a/F_y \leq 0.16$ $675/\sqrt{F_y}$	
	h/t_w	--	$1995/\sqrt{F_y}$
Circular hallow sections in axial compression	D/t	$22{,}750/\sqrt{F_y}$	--
Circular hallow section in flexure	D/t	$22{,}750/\sqrt{F_y}$	--
Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios	
		Compact	Non – Compact
Stems of tees	d/t	NA	$333/\sqrt{F_y}$
Unstiffened elements simply Supported along one edge, such as legs of single-angle struts, legs of double angle struts with separators and cross or star-shaped cross sections	b/t	NA	$200/\sqrt{F_y}$
Flanges of square and rectangular box and hallow structural sections of uniform thickness subject to bending or compression; flange cover plates and diaphragm plates between lines of fasteners or welds.	b/t	$500/\sqrt{F_y}$	$625/\sqrt{F_y}$

[a] For hybrid beams, use the yield strength of the flange F_{yf} instead of F_y

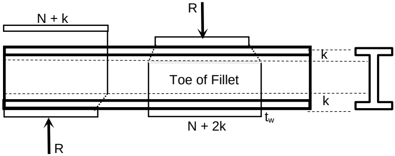
[b] Assumes net area of plate at widest hole.

[c] For design of slender section that exceed the non-compact limits See Section 502.6.2.2.

[d] See also Section 506.4.1

[e] $k_c = 4.05/(h/t)^{0.46}$ if h/t > 70, otherwise $k_c = 1.0$

WEB CRIPPLING



1992 NSCP
For Interior Loads

$$\frac{R}{(N+2k)\, t_w} \le 0.75\ F_y \qquad \text{Eq. 3 – 56}$$

For End Reaction

$$\frac{R}{(N+k)\, t_w} \le 0.75\ F_y \qquad \text{Eq. 3 – 57}$$

Where N = bearing length
t_w = thickness of web
k = distance from outer face of flange to toe of fillet

2001 NSCP

Local Web Yielding

Bearing stiffeners shall be provided if the compressive stress at the web toe of the fillets resulting from concentrated loads exceeds 0.66 F_y.

When the force to be resisted is a concentrated load producing tension or compression, applied at a distance from the member end that is greater than the depth of the member:

$$\frac{R}{(N+5k)\,t_w} \leq 0.66\,F_y \qquad \text{Eq. 3 – 58}$$

When the force to be resisted is a concentrated load applied at or near the end of the member:

$$\frac{R}{(N+2.5k)\,t_w} \leq 0.66\,F_y \qquad \text{Eq. 3 – 59}$$

Web Crippling

Bearing stiffeners shall be provided in the webs of members under concentrated loads, when compressive force exceeds the following limits:

When the concentrated load is applied at a distance not less than d/2 from the end of the member:

$$R=177.2\,t_w^{2}\left[1+3\,\frac{N}{d}\,\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{F_{yw}\;t_f/t_w} \qquad \text{Eq. 3 – 60}$$

When the concentrated load is applied less than distance d/2 from the end of the member:

$$R=89.3\,t_w^{2}\left[1+3\,\frac{N}{d}\,\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{F_{yw}\;t_f/t_w} \qquad \text{Eq. 3 – 61}$$

Where F_{yw} = specified minimum yield stress of beam web MPa

d = overall depth of the member, mm

t_f = flange thickness, mm

N = bearing length (not less than N for end reactions)

BEARING PLATES

Masonry Bearing (Sect. 4.5.5)

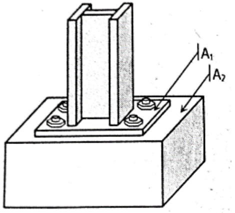
In the absence of Code regulation the following stresses apply:

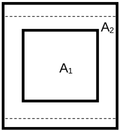
On sandstone and limestone……………….F_p = 2.76 MPa

On brick in cement mortar…………………..F_p = 1.75 MPa

On the full area of concrete support……….$F_p = 0.35\ f_c$

On less than the full area of concrete support……………………….. $F_p = 0.35\ f_c \sqrt{A_2/A_1} \leq 0.7\ f'_c$





Where f'_c = specified compressive strength of concrete, MPa

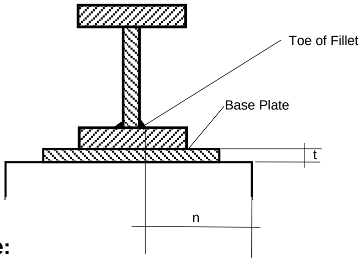
A_1 = area of steel concentrically bearing on a concrete support, mm^2

A_2 = maximum area of the portion of the supporting surface of concrete that is geometrically similar to and concentric with the loaded area, mm^2

Allowable Bending Stress in Steel Plate

$$F_p = 0.75\, F_y \qquad \text{Eq. } 3-62$$

Beam Base Plate



Thickness of plate:

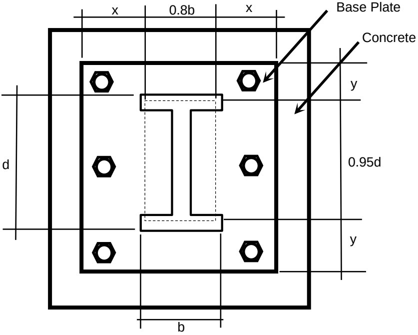
$$t = \sqrt{\frac{3\, f_p\, n^2}{F_b}} \qquad \text{Eq. } 3-63$$

$$f_b = \frac{\text{Load}}{\text{Bearing Area of Plate}} \qquad \text{Eq. } 3-64$$

Where f_p = actual bearing stress

F_b = allowable bending stress of plates = 0.75 F_y

Column Base Plate



Thickness of plate:

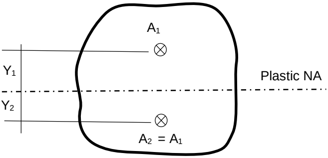
$$t = \sqrt{\frac{3\, f_n\, n^2}{F_b}} \qquad \text{Eq. 3 – 65}$$

Where n is the larger value of x and y

PLASTIC ANALYSIS AND DESIGN

Plastic Neutral Axis

The plastic neutral axis of a section is the line that divided the section into two equal areas.



Plastic Section Modulus

$$Z = \sum Ay = A_1\, y_1 + A_2\, y_2 + \ldots \qquad \text{Eq. 3 – 66}$$

For rectangular section, $Z = \frac{bd^2}{4}$ Eq. 3 – 67

Plastic Moment Capacity

$$M_p = F_y Z \qquad \text{Eq. 3 – 68}$$

Shapes Factor

$$SF = \frac{Z}{S} = \frac{M_P}{M_E} \qquad \text{Eq. 3 – 69}$$

For rectangular section, SF = 1.5

Where: S = elastic section modulus

M_E = elastic moment capacity

$$M_e = \frac{F_y I}{c} = \frac{F_y}{S}$$

TIMBER DESIGN

BASIC CODE REQUIREMENTS

Definition

Blocked Diaphragm is diaphragm in which all sheathing edges not occurring on framing members are supported on framing members are supported on and connected to blocking.

Convention Light-Frame Construction is a type of construction whose primary structural elements are formed by a system of repetitive wood-framing members.

Diaphragm is a horizontal or nearly horizontal system acting to transmit lateral forces to the vertical resisting elements when the term diaphragm is used, it includes horizontal bracing systems.

Fiberboard is a fibrous-felted homogeneous panel made from lignocellulosic fibers (usually wood or crane) having a density of less than 497kg/m3 but more than 160 kg/m3.

Glued Built-Up Members are structural elements, the sections of which are composed of built-up lumber, wood structural panels or wood structural panels in combination with lumber, all parts bonded together with adhesive.

Grade (Lumber), the classification of lumber in regard to strength and utility in accordance with the grading rules of an approved lumber grading agency.

Hardboard is a fibrous-felted, homogeneous panel made from lignocellulosic fibers consolidated under heat and pressure in a hot press to a density not less than 497 kg/m3.

Nominal Size (Lumber), the commercial size designation of width and depth, in standard sawn lumber grades, somewhat larger than the standard net size of dressed lumber. In accordance to Philippine National Standards (PNS).

Normal Loading, a design load that stressed a member or fastening to the allowable stress tabulated in this chapter. This loading may be applied for the remainder of the life of the member or fastening.

Particleboard is a manufactured panel product consisting of particles of wood fibers bonded together with synthetic resins or other suitable bonding system by as bonding process, in accordance with approved nationally recognized standard.

Plywood is a panel of laminated veneers conforming to Philippine National standards (PNS) "Construction and Industrial Plywood" and UBS Standard 23-3, "Performance Standard for Wood-based Structural-User Panels".

Rotation is the torsional movement of a diaphragm about a vertical axis.

Sub diaphragm is a portion of a larger wood diaphragm designed to anchor and transfer local forces to primary diaphragm struts and the main diaphragm.

Treated Wood, is wood treated with an approved preservative under treating and quality control procedures.

Wood on Natural Resistance to decay or Termites is the heartwood of the species set forth below. Corner sapwood is permitted on 5 percent of the pieces provided 90 percent or more of the width of each side on which it occurs is heartwood. Recognized species are:

Decay resistant: Narra, Kamagong, Dao, Tangile.

Termite resistant: Narra, Kamagong.

Wood Structural Panel is a structural panel product composed primarily of wood and in meeting the requirements of the Philippine National Standards (PNS). Wood structural panels include all veneer plywood, composite panels containing a combination of veneer and wood-based material and mal-formed panel such as oriented stranded board and wafer board.

Duration of Load

Values for wood and mechanical fastenings (when the worst determines the load capacity) are subjected to the following adjustments for the various duration of loading.

1. Where a member is fully stressed to the maximum allowable stress either continuously or cumulatively for more than 10 years under the conditions of maximum design load, the values shall not exceed 90 percent of those in the tables.
2. When the accumulated duration of the full maximum load during the life of the member does not exceed the period indicated below, the values may be increased in the table as follows:

   25% for seven days duration, as for roof loads
   33.33% for earthquake
   33.33% for wind (for connections and fasteners)
   60% for wind (members only)
   100% for impact

   The foregoing increases are not cumulative. For combined duration of loadings the resultant structural members shall not be smaller than the required for the longer duration of loading.

   The duration of load factors in this item shall not apply to compression-perpendicular-to-grain design values based on a deformation limit, or to modulus of elasticity.

3. Values for normal loading conditions may be used without regard to impact if the stress induced by impact does not exceed the values for normal loading

Size Factor Adjustment

When the depth of a rectangular sawn lumber bending member 125mm or thicker exceeds 300mm, the bending values, F_{b1} shall be multiplied by the size factor, C_1 as determined by:

$$C_f = \left(\frac{300}{d}\right)^{\frac{1}{9}} \qquad \text{Eq. } 4-1$$

Where C_F = size factor

d = depth of beam in mm

For beams of circular cross section that have a diameter greater than 340mm, 300mm or larger square beams loaded in plane of the diagonal, the size factor C_F may be determined on the basis of an equivalent conventionally loaded square beam of the same cross-sectional area.

Size factor adjustments are cumulative with form factor adjustments, except for lumber I-beam and box-beams, but are not cumulative with slenderness factor adjustments. The size factor adjustment shall not apply to visually graded lumber 50mm to 100mm thick or to machine- stress-rated lumber.

Slenderness Factor

When the depth of a bending member exceeds its breadth lateral support may be required and the slenderness factor C_s shall be calculated by:

$$C_s = \sqrt{\frac{l_e\,d}{b^2}} \qquad \text{Eq. } 4-2$$

Where C_s = slenderness factor
l_e = effective length of beam, mm from Table 4-1
d = depth of beam, mm
b = breadth of beam, mm

Table 4 – 1: Effective Length of Beams

Type of Beam Span and Nature of Load	L
Single-span beam, load concentrated at the center	1.61 L_u
Single-span beam, uniformly distributed load	1.92 L_u
Single-span beam, equal end moments	1.84 L_u
Cantilever Beam, Load concentrated	1.06 L_u
Cantilever Beam, Load concentrated at unsupported end	1.69 L_u
Cantilever beam, uniformly distributed load	1.69 L_u
Cantilever beam, uniformly distributed load with concentrated load at cantilever end	1.92 L_u
L_u = unsupported length of beam, mm	

The effective lengths l_e in Table 4-1 are based on L_u/d ratio of 17. For other L_u/d ratios, these effective lengths may be multiplied by a factor equal to $0.85 + 2.55/(L_u/d)$ except that this factor shall not apply to a single –span beam with equal end moments ($l_e = 1.84 L_u$) or to a single span or cantilever with any load ($l_e = 1.92 L_u$).

Unsupported Length, L_u

When the compression edge of a beam is supported throughout its length to prevent its lateral displacement, and the end at points of bearing have lateral support to prevent rotation, the unsupported length L_u may be taken as zero.

When lateral support is provided to prevent rotation at the points of end bearing but no other lateral support is provide throughout the length of the beam, the unsupported length L_u is the distance between such points of end bearing or the length of the cantilever.

When a beam is provided with a lateral support to prevent rotational and lateral displacement at intermediate points as well as the ends, unsupported length L_u is the distance between such points of intermediate lateral support.

FLEXTURAL STRESS

When $C_s \leq 10$

When the slenderness factor C_s does not exceed 10, the full allowable unit stress in bending F_b may be used.

When $C_s > 10$ and $C_s \leq C_k$

When the slenderness factor C_s is greater than 10 but does not exceed C_k the allowable unit stress in bending F'_b shall be:

$$F'_b = F_b \left[1 - \frac{1}{3}\left(\frac{C_s}{C_k}\right)^4\right] \qquad \text{Eq. 4 – 3}$$

When $C_s > C_k$ and $C_s < 50$

When the slenderness factor C_s is greater than C_k but less than 50, the allowable unit stress in bending F'_b shall be:

$$F'_b = \frac{0.438\ E}{C_s^{\ 2}} \qquad \text{Eq. 4 – 4}$$

In no case shall C_s exceed 50.

Where: $C_k = 0.811\sqrt{E/F_b}$

E = modulus of elasticity

F_b = allowable unit stress for extreme fiber in bending

F'_b = allowable unit stress for extreme fiber in bending, adjusted for slenderness.

<u>Form Factor Adjustments (for non-prismatic members)</u>

The allowable unit stress in bending for non-prismatic members shall not exceed the value established by multiplying such stress by the form factor C_f determined as follows:

BEAM SECTION	**FORM FACTOR (C_f)**
Circular	1.180
Square (with diagonal vertical)	1.414

Lumber I Beams and Box Beam	$0.81\left\{1+\left[\frac{\left(\frac{d}{25.4}\right)^2+143}{\left(\frac{d}{25.4}\right)^2+88}-1\right]C_s\right\}$ Eq. 4 – 5

$$C_g = p^2\left(6 - 8p + 3p^2\right)(1 - q) + q \qquad \text{Eq. 4-6}$$

Where C_f = form factor

C_g = support factor

p = ratio of depth of compression flange to full depth of beam

q = ratio of thickness of web or webs to the full width of beam

The form factor adjustment shall be cumulative with the size factor adjustment, except for the lumber l Beams and Box Beams.

Modulus of Elasticity Adjustment

The use of average modulus of elasticity values are appropriate for the design of normal wood structural members and assemblies. In special applications where deflections critical to the stability of structures or structural components, and where exposed to varying conditions, the average the average values of the modulus of elasticity E for lumber as follows:

Visually graded sawn lumber, $C_v = 0.25$

Machine stress-rated sawn lumber, $C_v = 0.11$

The average modulus of elasticity E values listed in the Table shall be multiplied by $1 - C_v$, or $1 – 1.65\ C_v$ to obtain a modulus of elasticity E value exceeded by 84 percent or 95 percent individual pieces, respectively.

DESIGN OF HORIZONTAL MEMBERS

Beam Span

For simple beams, the span shall be taken as the distance from face to face of support, plus one half the required length of bearing at each end; for continuous beams, the span is the distance between centers of bearings on support over which the beam is continuous

Flexure

Circular Cross Section

A beam of circular cross section may be assumed to have the same strength in flexure as a square beam having the same cross-sectional area. If a circular beam is tapered, it shall be considered a beam of variable cross section

Notching

If possible, notching of beams should be avoided. Notches in sawn lumber bending members shall not exceed one-sixth the depth of the member and shall not be located in the middle third of the span. Where members are notches at the ends, the notch depth shall not exceed one-fourth the beam-depth. The tension side of the sawn lumber bending members of 100mm or greater nominal thickness shall not be notched except at ends of members. Cantilevered portions of beams less than 100mm in normal thickness shall not be notched unless the reduced section properties and lumber defects are considered in the design.

Horizontal Shear

The maximum horizontal shear stress in a solid-sawn wood shall not exceed:

$$f_v = \frac{3V}{2bd} \qquad Eq. 4-7$$

The actual unit shear f_v shall not exceed the allowable for the species and the grade as given in Table 4-3 adjusted for duration of loading.

When calculating the shear force, V, distribution of load to adjacent parallel beams by flooring or other members may be considered, and all loads within a distance from either support equal to the depth of the beam may be neglected for beams support by full bearing on one surface and loads applied to the opposite surface.

Horizontal Shear in Notched Beams

When rectangular-shaped girder, beams or joists are notched at points of support on the tension side, they shall meet the design requirements of that section in bending and in shear. The horizontal shear stress at such point shall not exceed:

$$f_v = \frac{3V}{2bd}\left(\frac{d}{d'}\right) \qquad Eq. 4-8$$

Where: d = total depth of beam

d' = actual depth of beam of notch

When girder, beams or joists with circular cross section are notched at points of support on the tension side, they shall meet design requirements of that section in bending and in shear. The actual shear stress at such point shall not exceed:

$$f_v = \frac{3V}{2\ A_n}\left(\frac{d}{d_n}\right) \qquad Eq.4-9$$

Where: A_n = cross sectional area of notched member

d = total depth of beam

d' = actual depth of beam of notch

When girder, beams or joists with circular cross section are notched at points of support on the compression side, they shall meet design requirement for that net section in bending and in shear. The requirement for that net section in bending and in shear. The shear at such point shall not exceed the value calculated by

$$V= \frac{2}{3}\ F_v\ b\left[d - \frac{d\text{-}d'}{d'}\ e\right] \qquad Eq.4-10$$

Where: d = total depth of beam

d' = actual depth of beam of notch

e = distance notch extends inside the inner edge of support

The shear for the notch on the compression side shall be further limited to the value determined for a beam of depth d' if e exceeds d'.

<u>Design of Joints in Shear</u>

Eccentric connector and bolted joints and beams support by connectors or bolt shall be designed so that f_v in Eq. 4-11 does not exceed the allowable unit stresses in horizontal shear.

$$f_v = \frac{3V}{2bd_e} \qquad Eq.4-11$$

Where: d_e (with connectors) = the depth of the member less the distance from the unloaded edge of the member to the nearest edge of the nearest connector

d_e (with bolts or lag screws) = the depth of the member less the distance from the unloaded edge of the member to the nearest edge of the member to the center of the nearest bolt or lag screw.

Allowable unit stresses in shear for joint involving bolts or connectors loaded perpendicular to grain may be 50 percent greater than the horizontal shear values as set forth in Table 4-3 and, provided that the joint occurs at least five times the depth of the member from its end, the included shear stress is calculated by:

$$f_v = \frac{3V}{2bd_e}\left(\frac{d}{d_e}\right) \qquad Eq.4-12$$

and the 50 percent increase in design values for shear in joints does not apply.

Compression Perpendicular to Grain

In application where deformation is critical, Eq. 4-13 shall be used to calculate the compression- perpendicular-to-grain design values.

$$F_{c\perp}' = 0.73\, F_{c\perp} \qquad Eq. 4-13$$

Where: $F_{c\perp}$ = compression perpendicular-to-grain values from Table 4-3

$F_{c\perp}'$ = critical compression-perpendicular-to-grain value

For bearing less than 150mm in length and not nearer than 75mm to the end of a member, the maximum allowable load per square mm may be obtained by multiplying the allowable unit stresses in compression perpendicular to grain factor given by:

$$C_b = \frac{l_b + 9.5}{l_b} \qquad Eq. 4-14$$

where l_b is the length of bearing in mm measured along the grain of the wood.

The multiplying factors for indicated length of bearing on such small areas plates and washers may be:

Length of bearing (mm)	**13**	**25**	**38**	**50**	**75**	**100**	**150 or more**

Factor	1.75	1.38	1.25	1.19	1.13	1.10	1.00

In using the preceding equation and table for round washers or bearing areas, use length equal to the diameter.

Lateral support

Solid-sawn rectangular lumber beams, rafter and joist shall be supported laterally to prevent rotation or lateral displacement in accordance with the following:

If the ratio of depth to thickness, based on nominal dimensions, is:

1. Two to 1, no lateral support is required.
2. Three to 1 or 4 to 1, the ends shall be held in position, as by full-depth solid blocking, bridging, nailing or bolting to other framing members, approved hangers or other framing members, approved hangers or other acceptable means
3. Five to 1, one edge shall be held in line for its entire length.
4. Six to 1, bridging, full-depth solid blocking or cross bracing shall be installed at intervals not exceeding 2.4 meters unless
   - 4.1 Both edges of the member are held in line or,
   - 4.2 The compression edge of the member is supported throughout its length to prevent lateral displacement, as by adequate sheathing or sub flooring, and the ends and all points of bearing have lateral support to prevent rotation.

5. Seven to 1, both edges shall be held in line for their entire length.

If a beam is subject to both flexure and compression parallel to grain, the ratio maybe as much as 5 to 1 if one edge is held firmly inline. If under any combination of load the unbraced edge of the member is in tension, the ratio may be 6 to 1.

COLUMN DESIGN

Column Classifications

Simple Solid-Wood Columns

Simple column consist of a single piece or of pieces properly glued together to form a single member.

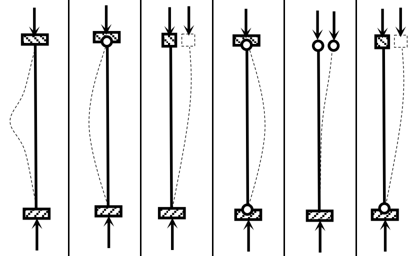
Spaced Column, Connector Joined

Spaced columns are formed of two or more individual members with their longitudinal axes parallel, separated at the ends and middle points of their length by blocking and joined at the ends by timber connectors capable of developing the required shear resistance.

Simple Solid-Column Design

The effective column length l_e shall be used in design Equations given in this section. The effective column length, l_e shall be determined in accordance with good engineering practice. Actual column length, l, may be multiplied by the factors given in Table 4-2 determine effective column length l_e ($l_e = K_e\, l$)

Table 4 – 2 Effective Length factors, K_e

Buckled shape of column is shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.20	1.00	2.10	2.00
End conditions code		Rotation fixed and translation fixed				
		Rotation free and translation fixed				
		Rotation fixed and translation free				
		Rotation free and translation free				

Allowable unit stresses in N per mm^2 of cross-sectional area of square or rectangular simple solid columns shall be determined by the following formulas, but such unit stresses shall not exceed values for compression, parallel to grain F_c in Table 4-3 adjusted in accordance with provision of this section.

$$F'_c = F_c^* \left[\frac{1+\alpha}{2c'} - \sqrt{\left(\frac{1+\alpha}{2c'}\right)^2 - \frac{\alpha}{c'}} \right] \qquad \text{Eq. 4-15}$$

Where $\alpha = \frac{F_{cE}}{F_c^*}$

c' = 0.8 for sawn lumber & 0.85 for round timber pile

$F_{cE} = \frac{K_{cE}\,E'}{(l_e/d)^2}$

F_c^* = tabulated compression design value multiplied by all of the applicable adjustment factors

K_{cE} = 0.3 for visually graded lumber

K_{cE} = 0.418 for products such as machine stress rated sawn lumber

Tapered Columns

When designing a tapered column with a rectangular cross-section, tapered at one end or both ends, the representative dimensions, d_{rep} for each face of the column shall be:

$$d_{rep} = d_{min} + \left(d_{max} - d_{min}\right)\left[a - 0.15\left(1 - d_{min}/d_{max}\right)\right] \qquad \text{Eq. 4–16}$$

Where: d_{min} = minimum dimension for that face of the column

d_{max} = maximum dimension for that face of the column

Support conditions:

Large end fixed, small end unsupported.

a=0.70

Small end fixed, large end unsupported.

a=0.30

Both ends supported:

Tapered toward one end.

a=0.50

Tapered toward both ends. A=0.70

For all other support conditions

$$d = d_{min} + \frac{d_{max} - d_{min}}{3} \qquad Eq. 4-17$$

The design of a column of round cross-section shall be based on the design calculation for a square column of the same cross-sectional area and having the same degree of taper.

COMBINED FLEXURE AND AXIAL LOAD

Flexure and Axial Tension

Members subjected to both flexure and axial tension shall be so proportioned that

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^{*}} \leq 1.0 \qquad Eq. 4-18$$

$$\text{and } \frac{f_b - f_t}{F_b^{**}} \qquad Eq. 4-19$$

Where: F_b^* = tabulated bending design value multiplied by all applicable adjustment factors except C_F

F_b^{**} = tabulated bending design value multiplied by all applicable adjustment factor except C_F

f_t' = allowable tension design value parallel to grain.

f_b = actual unit stress for extreme fiber in bending

Flexure and Axial Compression

Members subjected to both flexure and axial compression shall be proportioned that

$$\frac{f_c}{F'_c} + \frac{f_{bx}}{F'_{bx} - Jf_c} \leq 1.0 \qquad \text{Eq. 4-20}$$

$$J = \frac{l_e/d - 11}{K - 11} \qquad Eq. 4 - 21$$

$$K = 0.671\sqrt{\frac{E}{F_c}} \qquad Eq. 4 - 22$$

Where $0 \leq J \leq 1.0$

F'_c and K shall be determined in accordance with Eq. 4-15 except (1) when checking the design in the plane of bending the slenderness ratio, l_e/d, in the plane of bending shall be used to perpendicular to the plane of bending shall be used to calculate F'_c and J shall be set equal to zero.

Spaced columns

In the case of spaced columns, this combined stress formula maybe applied only if the bending is in a direction parallel to the greater d of the individual member.

Truss Compression Chords

Effect of buckling of a 50mm by 100mm or smaller truss compression chord having effective buckling lengths of 2.40 m or less and with 9 mm or thicker plywood sheathing nailed to the narrow face of the chord in accordance with the appropriate standards shall be determined from the formula

$$C_T = \frac{1 + 0.62\ I_e}{E_{0.05}} \qquad Eq. 4-23$$

Where: C_T = buckling of the stiffness factor

C_T = 0.819E for machine-stress-rated lumber

I_e = effective buckling length used in design of chord for compression loading

$E_{0.05}$ = 0.589E for visually graded lumber

E = Modulus of elasticity from Table 4-3, MPa

The values of C_T is determined from this equation are for wood seasoned to a moisture content of 19 percent or less at the time the plywood is nailed to the chord . For wood that is unseasoned at the time of plywood attachment, C_T shall be determined from Eq. 4-24:

$$C_T = \frac{1 + 0.33\, l_e}{E_{0.05}} \qquad Eq.4-24$$

For chords with an effective buckling length greater than 2.40 m, C_T shall be taken as the value for a chord having an effective length of 2.40 m.

The buckling stiffness factor does not apply to short columns or trusses used under wet conditions. The allowable unit compressive stress shall be modified by the buckling stiffness factor when a truss chord is subjected to combined flexure and compression and the bending moment in the direction that induces compression stresses in the chord face to which the plywood is attached.

The buckling stiffness factor C_T shall apply as follows:

Short column ($l_e/d \leq 11$)

$$F'_c = F_c \qquad Eq.4-25$$

Intermediate column ($11 < l_e/d < K$)

$$K = 0.671 \sqrt{C_T \frac{R}{F_c}} \qquad Eq.4-26$$

$$F'_c = F_c \left[1 - \frac{1}{3}\left(\frac{l_e/d}{K}\right)^4\right] \qquad Eq.4-27$$

Long column ($l_e/d \geq K$)

$$F'_c = \frac{0.30\ E\ C_T}{(l_e/d)^2} \qquad Eq. 4-28$$

Compression at an Angle to Grain

The allowable unit stress in compression at an angle of load to grain between 0° to 90° shall be computed from the Hankinson's Equation as follows:

$$F_n = \frac{F_c F_{c\perp}}{F_c \sin^2\theta + F_{c\perp}\cos^2\theta} \qquad Eq. 4-29$$

F_c shall be adjusted for duration of load before use in Hankinson's Formula. Values of F_n and $F_{c\perp}$ are not subjected to duration of load modifications.

TIMBER CONNECTORS AND FASTENERS

Timber connectors and fasteners may be used to transmit forces between wood members and between wood and metal members. The allowable loads and installation of timber connectors and fasteners shall be in accordance with the tables as provided in this Chapter.

Bolts

Safe loads in kN for bolts in shear in seasoned lumber shall not exceed the values set forth in Table 4-4.

Allowable shear values used to connect a wood to concrete or masonry are permitted to be determined as one half the tabulated double shear values for a wood member twice the thickness of the member attached to the concrete or masonry.

The loads given in Table 4-4 are for a joint consisting of three members. The bolts are in double shear. The length of the bolt l, is the thickness of the main member.

NAILS AND SPIKES

Safe lateral strength

A common wire nail driven perpendicular to grain of the wood. When used to fasten wood members together, shall not be subjected to a greater load causing shear and bending than the safe lateral strength of the wire nail or spike as set forth in the Table.

A wire nail driven parallel to the grain of the wood shall not be subjected more than two thirds of the lateral load allowed when driven perpendicular to the grain. Toenails shall not be subjected more than five sixths of the lateral load allowed for nails driven perpendicular to the grain.

Safe resistance to withdrawal

A wire nail driven perpendicular to grain of wood shall not be subjected to a greater load, tending to cause withdrawal than the safe resistance of the nail to withdrawal as set forth in the Table.

Spacing and penetration

Common wire nails shall have penetration into the piece receiving the point as set forth in the Table. Nails or spikes for which the wire gauges or lengths are not set forth in the Table shall have a required penetration of not less than 11 diameters, and allowable loads may be interpolated. Design values shall be increased when the penetration of nails into the member holding the point is larger than the required by this item.

For wood-to-wood joints, the spacing center to center of the nails in the direction of stress shall not be less than one half of the required penetration. Edge or end distances in the direction of stress shall not be less one half of the required penetration. All spacing and edge and end distances shall be such as to avoid splitting of the wood.

Holes for nails, where necessary to prevent splitting, shall be bored of a diameter smaller than that of the nails
Joist hangers and Framing Anchors
Connections depending upon joist hangers or framing anchors, ties and other mechanical fastenings not otherwise covered may be used where approved.

Miscellaneous Fasteners

Drift bolts or Drift pins

Connections involving the use of drift bolts or pins, wood and screws and lag screws shall be designed in accordance with the provision set forth in this chapter

Withdrawal Design Values

Drift bolt and drift pin connections loaded in withdrawal shall be designed in accordance with good engineering practice.

Lateral Design Values

Allowable lateral design values for drift bolts and drift pins driven in the side grain of wood shall not exceed 75 percent of the allowable lateral design values for common bolts of the same diameter and length in main member. Additional penetration of pin into members should be provided in lieu of the washer, head and nut on a common bolt.

Spike grids

Wood-to-wood connections involving spike grids for lateral load transfer shall be designed in accordance with good engineering practice.

SURVEYING AND TRANSPORTATION ENGINEERING

UNITS OF MEASUREMENT

Most-used Equivalents in Survey Works:

1 rod = 1 pole = 1 perch = 16.5 ft
1 engineer's chain = 100 ft = 100 links
1 Gunter's chain = 66ft
= Gunter's links (lk)
= 4 rods = $\frac{1}{80}$ mile
1 acre = 100,000 sq. (Gunter's) links = 43, 560 ft^2
1 rood = $^1/_4$ acre = 40 $rods^2$
1 hectare = 10,000 m^2 = 2.471 acres
1 arpent = about 0.85 acre
1 statute mile = 5280 ft = 1609.35 m
1 mi^2 = 640 acres
1 nautical mile = 6080.27 ft = 1853.248 m
1 fathom = 6 ft
1 cubit = 18 in
1 vara = 33 in
1 degree = $\frac{1}{360}$ circle = 60 min = 3600 s
1 grad (grade) = $\frac{1}{400}$ circle = $\frac{1}{100}$ quadrant
1 mil = $\frac{1}{6400}$ circle = $\frac{1}{100}$ quadrant
1 military pace = 2.5 ft

THEORY OF ERRORS

When a number of measurement of the same quantity have been made, they must be analyzed on the basis of probability and the theory of errors.

After all systematic (cumulative) errors and mistakes have been eliminated random (compensating) errors are investigated to determine the most probable value (mean) and other critical values.

MOST PROBABLE VALUE

$$\boxed{\text{mpv}, \ \bar{X} = \frac{\sum X}{n}}$$

Where : $\sum X$ = sum of all individual measurements
n = total number of measurements made

Residual, v

Residual or deviation is the difference between any measured value of a quantity and its most probable value,

$$\boxed{v = X - \bar{x}}$$

PROBABLE ERROR

The probable error is a quantity which, when added to and subtracted from the most probable value, defines range within which there is fifty percent chance that the true value of the measured quantity lies inside (or outside) the limits this set.

Probable Error of any Single Measurement

$$PE_s = \pm 0.6745 \sqrt{\frac{\sum v^2}{n-1}}$$

Probable Error of the Mean:

$$PE_m = \pm 0.6745 \sqrt{\frac{\sum v^2}{n\,(n-1)}}$$

Where n = number of observations

$\sum v^2$ = summation of the squares of the residuals

INTERRELATIONSHIP OF ERRORS

Sum of Errors

Probable error of the sum

$$PE_s = \pm\sqrt{PE_1^{\,2} + PE_2^{\,2} + \ldots + PE_n^{\,2}}$$

Where PE_1, etc = probable error of each measurement

Product of Errors

Probable error of the product

$$PE_s = \pm\sqrt{(Q_1\,PE_1)^2 + (Q_2\,PE_2)^2}$$

Where: Q_1, Q_2 = measured quantities
PE_1, PE_2 = probable error corresponding to each quantity measured.

Precision

$$\boxed{\text{Precision} = \frac{PE_m}{mpv}}$$

Standard Deviation

Example: Given the following data:

- 5, 7, 2, 3, 5, 2, 7, 2, 12

The **MEDIAN** is the middle value when all data are arranged in decreasing or increasing order

-5, 2, 2, 2, 3, 5, 7, 7, 12 → (9 terms)

The median is the 5^{th} term = 3

Note: When there is an even number of values, the median is defined as the mean (average) of the middle two values.

The **MODE** is the value that occurs most frequently. The value “2” occurs three times, therefore, is the mode.

The **RANGE** is the difference between the maximum and minimum values.

Range = 12-(-5)=17

The **VARIANCE** is defined by:

$$\boxed{\text{Variance} = \frac{\sum\left(X-\bar{X}\right)^2}{n} = \frac{\sum v^2}{n}}$$

The **STANDARD DEVIATION** is

$$\boxed{SD = \sqrt{\text{variance}} = \sqrt{\frac{\sum\left(X-\bar{X}\right)^2}{n}}}$$

Where: X = value of an observation

$\bar{X}$ = most probable value $= \sum\frac{X}{n}$

MEASUREMENT OF DISTANCE

Pacing

$$PF = \frac{\text{Distance measured}}{\text{Average pace}} \text{ m/pace}$$

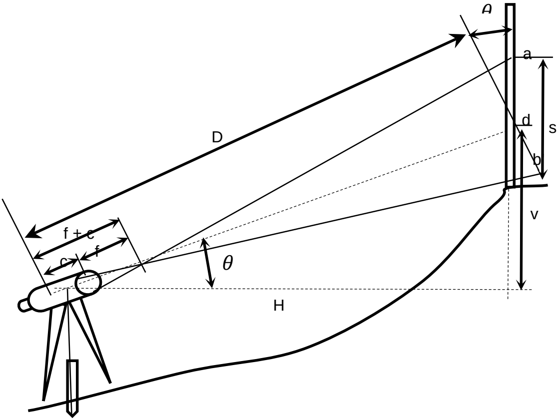
$$\text{Average pace} = \frac{\sum \text{Paces}}{\text{no.of observation}}$$

Stadia Measurement

$$D = f/i\ S\cos\theta + (f+c) = k\ S\cos\theta + (f+c)$$

$$H = D\cos\theta \qquad V = D\sin\theta$$

For horizontal line of sight $\theta = 0°$



Where:
f/I = k = stadia interval factor (equal to 100 for most instruments)
S = stadia constant
f + c = stadia constant (equal to zero for internal focusing)
θ = angle of inclination of the line of sight

$$\boxed{D = \frac{s}{2\tan\left(\frac{\theta}{2}\right)}}$$

For S = 2 m

$$\boxed{D = \frac{1}{\tan\left(\frac{\theta}{2}\right)} = \cot\frac{\theta}{2}}$$

S = length of bar (usually 2 m)
θ= angle subtended by the bar

CORRECTIONS IN TAPING

Temperature Change

$$\boxed{e = \alpha L_o (T - T_o)}$$

T = Temperature during measurement
T_o = Temperature when tape is length of L_o

α = coefficient of thermal expansion of tape, 11.6 x 10^{-6} /^oC for steel

Pull Correction

$$\boxed{e = \frac{(P - P_o) L_o}{A E}}$$

P = pull during measurement
P_o = pull when the tape is of length L_o
A = cross – sectional are of the tape
E = modulus of elasticity of the tape = 200 GPa for steel

Sag Correction (Negative Error)

$$e = \frac{w^2 L^3}{24 T^2}$$

W = weight of tape per linear m or ft
L = unsupported length of tape
T = pull during measurement

Slope Correction (Negative Error)

$$e = \frac{h^2}{2S}$$

h= difference in elevation between the ends of the tape
S = inclined distance

Corrected or True Distance of a Line

$$TD = MD + Error$$

TD = true or corrected distance of the line
MD = measured distance
Error = total error

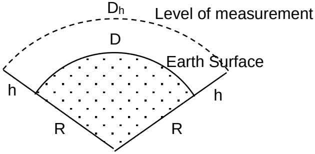
$$Error = e \times N$$
$$N = \frac{MD}{L_1}$$

e = error per tape length (+) if too long and (-) if too short

N = number of tape lengths
L_T = length of tape

Reduction to Sea Level

When horizontal measurement is done at high elevations the sea-level distance can be found by the following relationship:



By proportion of figures:

$$\frac{D}{R} = \frac{D_h}{R+h} \quad \text{or } D = \frac{D_h\, R}{R+h}$$

$$\text{Correction , } \Delta D = D_n - D$$

D_n = horizontal distance at an altitude of h (above sea level),
D = Actual or Corrected dist. At the surface of the earth
h = altitude of observation, m
R = radius of earth, m

Approximate formula

$$\text{Correction, } \Delta D = D_h\left(\frac{h^2}{R^2} - \frac{h}{R}\right)$$

$$\text{Reduction Factor, } k = 1 - \frac{h}{R}$$
$$D = D_h \times k$$

EFFECT OF EARTH'S CURVATURE AND REFRACTION

$$h_{cr} = 0.0675\ K^2$$
$$y = 1000\ K \tan\theta$$

where K is the distance of the parameter

LEVELING

$$HI = \text{Elev. of A} + BS$$
$$\text{Elev. of B} = HI - FS$$
$$\text{Difference in elev.} = FS - BS$$

Where: HI = height of instrument
BS = backsight
FS = foresight

SENSITIVITY OF BUBBLE TUBES

The division of bubble tubes are usually spaced at 2-mm intervals, the student would often wants to know how much the rod readings will be affected if the bubble were off center

$$\frac{\Delta h}{D} = \frac{c}{R}$$

Where: R = radius of curvature of the bubble tube
c = displacement of bubble from center, usually in number of spaces
D = horizontal distance from instrument to rod
$\triangle h$ = error in vertical reading

ERROR DUE TO NON-ADJUSTMENT OF TRANSIT TELESCOPE

The error in horizontal angle when the axis of the transit telescope is not horizontal.

$$\boxed{\text{Error, } E = e \tan\theta}$$

Where:
e = angle of inclination of the telescope axis, usually in minutes or seconds
θ = observed vertical angle of the object
E = error in horizontal angle in minutes or seconds

When two observations are made, the total error may be expressed as:

$$\boxed{\text{Error} = e\,(\tan\theta_1 - \tan\theta_2)}$$

Where:
θ_1= first vertical angle
θ_2 = second vertical angle

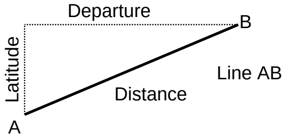
TRAVERSE

The survey procedure known as traversing is fundamental to much survey measurement. The procedure consists of using a variety of instrument combinations to create polar vectors in space that is ‘lines’ with a magnitude (distance) and direction (bearing). These vectors are generally contiguous and create a polygon which conforms to various mathematical and geometrical rules (which can be used to check the fieldwork and computations). The equipment used generally consists of something to determine direction like a compass or theodolite, and something to determine distance like a tape or Electromagnetic Distance Meter (EDM).

LATITUDE AND DEPARTURE OF A LINE

North Latitude is (+)
South Latitude is (-)

East Departure is (+)
West Departure is (-)



$$\text{Latitude} = \text{Distance} \times \cos\theta$$
$$\text{Departure} = \text{Distance} \times \sin\theta$$
$$\text{Distance} = \sqrt{(\text{Latitude})^2 + (\text{Departure})^2}$$

CLOSED TRAVERSE

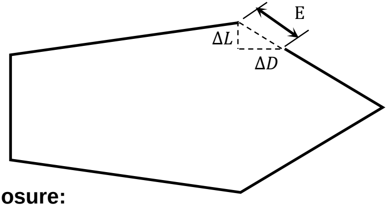
For a closed traverse,

$$\sum \text{North Latitude} = \sum \text{South Latitude}$$

$$\sum \text{East Departure} = \sum \text{West Departure}$$

ERROR OF CLOSURE

For any closed traverse where the north and south latitudes are not equal and not equal and the east and west departure are not equal



Error of Closure:

$$E = \sqrt{\Delta L^2 + \Delta D^2}$$

$$\text{Relative Error} = \frac{\text{Error of closure}}{\text{Perimeter of traverse}}$$

Angular Closure

The sum of the internal angles of a polygon (traverse) is given by the rule:

$$\boxed{\sum \alpha = 180° (n - 2)}$$

Where n is the number of sides of traverse, and α is each internal angle. Any variation from this sum is known as the misclosure and must be accounted for, either through compensation (if it is an acceptable amount) or elimination by repetition of the observations. An angular closure is computed for traverses performed with either theodolites or magnetic compasses. A larger misclosure could be expected when using a magnetic compass, but in any case it must be calculated and removed. The reduction of magnetic compass bearings to angles also eliminates the effect of local attraction.

BALANCING CLOSED TRAVERSE

Intuitive Method – The intuitive method is commonly used but difficult to explain. It is based on the Surveyor's understanding of the measurement process, and an acknowledgement that a line measured through dense bush in steep country is likely to have more accumulated random error than a line of similar length measured across flat grassy plains. Also, lines measure in the rain, after a pub lunch or just before quitting for the day may not be measured with the same degree of care as those at other times throughout the day. The Surveyor would perhaps add a few centimeters or so to one of suspect lines and recompute the misclosure.

Compass rule – (Bowditch Method) The correction to be applied to the latitude (or departure) of any course is to the total absolute correction in latitude (or departure) as the length of the course is to the perimeter of the traverse.

$$\frac{C_L}{|N_{Lat} - S_{Lat}|} = \frac{\text{Length of course}}{\text{Perimeter of Traverse}}$$

$$\frac{C_L}{|E_{Dep} - W_{dep}|} = \frac{\text{Length of course}}{\text{Perimeter of Traverse}}$$

Transit Rule – The correction to be applied to the latitude (or departure) of any course is to the total correction in latitude (or departure) as the latitude (or departure) of tat course is to the arithmetical sum of all latitudes (or departures) of the traverse.

$$\frac{C_L}{|N_{Lat} - S_{Lat}|} = \frac{\text{Latitude of the Course}}{N_{Lat} + |S_{Lat}|}$$

$$\frac{C_L}{|E_{Dep} - W_{dep}|} = \frac{\text{Latitude of the Course}}{E_{Dep} + |W_{Dep}|}$$

How to apply these Corrections

If the sum of the North Latitudes is greater than the sum of the South Latitudes, the correction is subtracted for North Latitudes and added for South Latitudes and vice versa.

If the sum of the East Departures is greater than the sum of the West Departures, the correction is subtracted for East Departures and added for West Departures and vice versa.

AREA OF CLOSED TRAVERSE

After balancing the traverse by applying either the compass rule or the transit rule, the area may be computed using the Double Meridian Distance (DMD) Method or the Double Parallel Distance (DPD) Method.

DMD Method

1. The DMD of the first course is equal to the departure of that course.
2. The DMD of any other course is equal to the DMD of the previous course plus the Departure of the course itself.
3. The DMD of the last course must be numerically equal to the departure of the last course but opposite in sign.
4. The double area of each course is equal to the product of the DMD and the Latitude of the course.

$$\boxed{\text{Double Area} = \text{DMD x Latitude}}$$

5. The area of the traverse is one-half the absolute value of the algebraic sum (consider the sign) of the double areas of all the courses.

$$\boxed{\text{Area} = \frac{1}{2}\left[\sum \text{Double Areas}\right]}$$

DPD Method

1. The DPD of the first course is equal to the departure of that course.
2. The DPD of any other course is equal to the DPD of the previous course plus the Latitude of the previous course plus the Latitude of the course itself.
3. The DPD of the last course must be numerically equal to the Latitude of the last course but opposite in sign.
4. The double area of each course but opposite in sign.

$$\text{Double Area} = \text{DPD} \times \text{Latitude}$$

5. The area of the traverse is one-half the absolute value of the algebraic sum (consider the sign) of the double areas of all the courses.

$$\boxed{\text{Area} = \frac{1}{2}\left[\sum \text{Double Areas}\right]}$$

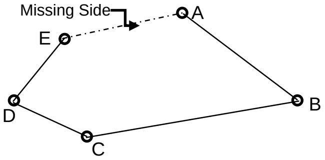
MISSING DATA

The missing elements of a traverse polygon that can be solved for are as follows:

1. Bearing and length of one side
2. Bearing of one side and length of adjacent side
3. Bearing of two adjacent sides
4. Bearing of two non-adjacent sides
5. Bearing of one side and length of one non-adjacent side
6. Length of two sides (adjacent or non-adjacent)

Only two missing elements can be determined as there are only two redundancies in a traverse network.

1. Bearing and Length of One Side:



This is the simplest of all cases because the unknown side is the closing line.

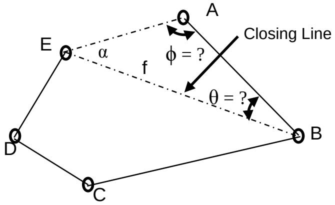
$$(\text{Lat})_{\text{missing side}} = -\sum \text{of latitudes of known sides}$$

$$(\text{Dep})_{\text{missing side}} = -\sum \text{of Departure of known sides}$$

$$\text{Distance} = \sqrt{(\text{Lat})^2 + (\text{Dep})^2}$$

$$\tan(\text{Bearing}) = \frac{\text{Dep}}{\text{Lat}}$$

2. Bearing of One Side and Length of Adjacent Side:



With reference to the figure, the missing data are length of side a, and side e.

With lengths and bearing of side b, c and d known, the closing line f can be solved. Angle α can be solved since the bearing of e is known.

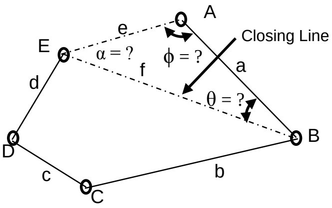
$$\frac{f}{\sin\phi} = \frac{a}{\sin\alpha} \;;\; \phi = \_\_\_\_\_\_\_$$

$$\theta + \alpha + \phi = 180° \;;\; \theta = \_\_\_\_\_\_\_$$

$$\frac{e}{\sin\theta} = \frac{a}{\sin\alpha} \;;\; e = \_\_\_\_\_\_\_$$

With angle ϕ and bearing of side e known, the bearing of side a can be determined.

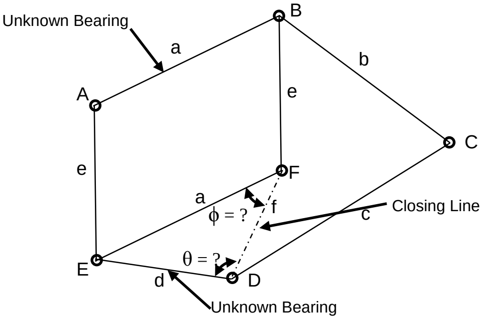
3. Bearing of Two Adjacent Sides:



With reference to the figure, the missing data are bearings of side e and side a.

With lengths and bearings of side b, c and d known, the closing line f can be solved.
With sides e, a and f known in triangle EAB, angles α and θ can be determined by cosine law and sine law.
With bearing of the closing line f known, the bearings of e and a can be determined.

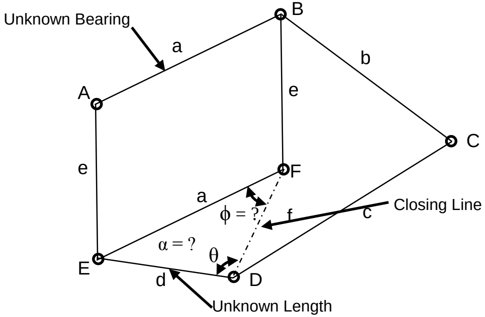
4. Bearing of Two Non – Adjacent Sides:



With reference to the figure, the bearings of side a and d are missing.

Shift side e to BF and solve the closing line f from polygon DCBF. Shift side a to EF to form triangle DEF with three known sides, and solve angles θ and ϕ. With bearing of closing side f known, the bearings of side a and d can be solved.

5. Bearing of One Side and Length of One Non – Adjacent Side:



With reference to the figure, the bearing of side a and length of side d are missing.

Shift side e to BF and solve the closing line f from polygon DCBF. Shift side a to EF to form triangle DEF.

With closing bearing of closing side f and side d known angle θ can be solve.

$$\frac{f}{\sin\alpha} = \frac{a}{\sin\theta} \ ; \ \alpha = \_\_\_\_\_\_\_$$

$$\theta + \alpha + \phi = 180° \ ; \ \phi = \_\_\_\_\_\_\_$$

$$\frac{d}{\sin\phi} = \frac{a}{\sin\theta} \ ; \ d = \_\_\_\_\_\_$$

With known ϕ and bearing of side f, the bearing of side a can be solved.

6. Length of Two Sides (Adjacent or Non – Adjacent)

Example: A closed traverse has the following data:

Line	Distance	Bearing
AB	179.00	N 47˚02'14" E
BC	258.20	S 69˚35'59" E
CD	?	S 39˚35'48" W
DE	?	S 87˚29'48" W
EA	145.41	N 24˚48'09" W

Find the lengths of CD and DE.

Solution:

Line	Distance	Bearing	Latitude	Departure
AB	179.00	N 47˚02'14" E	122	131
BC	258.20	S 69˚35'59" E	-90	242
CD	x	S 39˚35'48" W	-0.7706x	-0.6373x
DE	y	S 87˚29'48" W	-0.04368y	-0.999y
EA	145.41	N 24˚48'09" W	132	-61
			0	0

$\Sigma Lat = 122 - 90 - 0.7706x - 0.04368y + 132 = 0$

$17.642x + y = 3754.58$ (Eq. 1)

$\Sigma Dep = 131 + 242 - 0.6373x - 0.999y - 61 = 0$

$x + 1.5676y = 489.565$

$x = 489.565 - 1.5676y$ (Eq. 2)

$17.642(489.565 - 1.5676y) + y = 3754.58$

$8636.91 - 27.6556y + y = 3754.58$

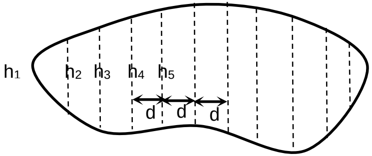
y = 183.16 m

x = 202.44 m

Note: This is also applicable if the unknown sides are non – adjacent.

AREA OF CROSS-SECTIONS AND VOLUME OF EARTHWORKS

The area of any irregular plane figure (such as the one shown) can be found approximately by dividing it into a number of strips or panels by a series of equidistant parallel chords (offsets) h_1, h_2, … h_n the common distance between the chords being d



AREA BY TRAPEZOIDAL RULE

Assuming each strip as a trapezoid, then area is:

$$\text{Area} = \frac{d}{2}[h_1 + 2(h_2 + h_3 + \ldots) + h_n]$$

AREA BY SIMPSON'S ONE-THIRD RULE

This method is more accurate than the previous because it considers the curved side. Using this rule, there must be an odd number of offsets, thus n must be odd.

$$\text{Area} = \frac{d}{2}\left[h_1 + 2\sum h_{odd} + 4\sum h_{even} + h_n\right]$$

AREA BY COORDINATES

The area of planar (convex or concave) with vertices

$$\text{Area} = \frac{1}{2}\left(\begin{vmatrix} X_1 & X_2 \\ Y_1 & Y_2 \end{vmatrix} + \begin{vmatrix} X_2 & X_3 \\ Y_2 & Y_3 \end{vmatrix} + \ldots + \begin{vmatrix} X_n & X_1 \\ Y_n & Y_1 \end{vmatrix}\right)$$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} X_1 & X_2 & X_3 & X_4 & \cdots & X_n & X_1 \\ Y_1 & Y_2 & Y_3 & Y_4 & \cdots & Y_n & Y_1 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2}\left[X_1Y_2 - X_2Y_1 + X_2Y_3 - X_3Y_2 + \ldots X_nY_1 - X_1Y_n\right]$$

The area of a polygon is defined to be positive if the points are arranged in a counterclockwise order and negative if they are in clockwise order

VOLUME BY END AREA METHOD

$$V_{\text{end area}} = \frac{A_1 + A_2}{2}\,L$$

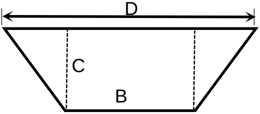
PRISMOIDAL FORMULA

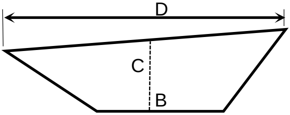
$$V = \frac{L}{6}\left(A_1 + 4\,A_m + A_2\right)$$

A_m = cross-sectional area at mid-section

PRISMOIDAL CORRECTION FORMULA

(Three-level section)



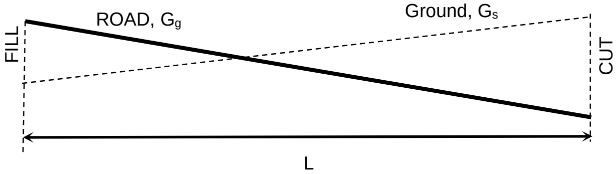


$$\boxed{V_{PC} = \frac{L}{12}\,(C_1 - C_2)(D_1 - D_2)}$$

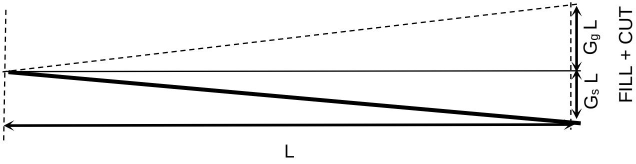
Corrected volume

$$\boxed{V_c = V_{endarea} - V_{PC}}$$

CUT AND FILL



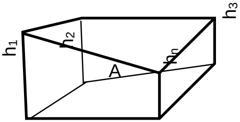
g



$$L = \frac{H_{CUT} + H_{FILL}}{G_{ground} - G_{road}}$$

VOLUME BY UNIT AREA METHOD

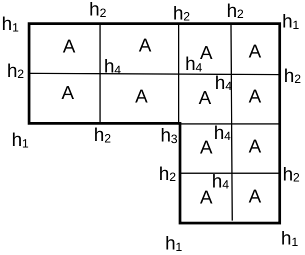
Truncated Prism



$$V = A\,\frac{\sum h}{n}$$

A = base area
h = corner height
n = number of corners

ASSEMBLY OF RECTANGULAR PRISM



$$V = A\,\frac{\sum h_1 + 2\sum h_2 + 3\sum h_3 + 4\sum h_4}{4}$$

h_1 = height found on one area only
h_2 = height common to two areas
h_3 = height found on three areas
h_4 = height common to four areas

VOLUME OF RESERVOIR OR PIT

Volume by End-Area Method

$$V = \frac{1}{2}\left[A_1 + 2\sum A_1 + A_n\right]$$

$\sum A_1$ = sum of areas of interior sections
A_1 & A_n = area of the first and last sections

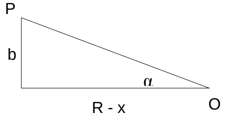
Volume by Prismoidal Formula

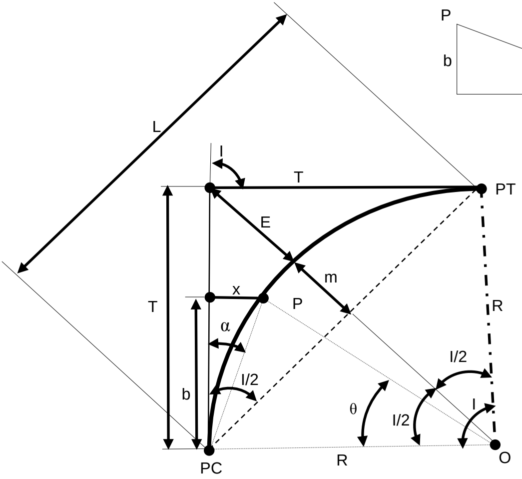
Applicable only for odd number of sections

$$\boxed{V = \frac{d}{3}\left[A_1 + 2\sum A_{odd} + 4\sum A_{even} + A_n\right]}$$

Note: if there is an even number of sections, use the end area to get the volume of the last segment

SIMPLE CURVE





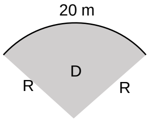
x = perpendicular offset from the tangent to the curve

α= deflection angle from PC to any point P on the curve

$$\boxed{\begin{gathered}\alpha = \frac{\theta}{2} \\ \tan\alpha = \frac{x}{b}\end{gathered}}$$

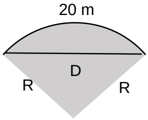
I = angle on intersection of the tangents or central angle of simple curve
D = degree of curve
PI = point of intersection (of tangents)
PC = point of curvature
PT = point of tangency

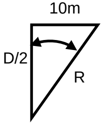
Radius of Curve
Arc Basis



$$20 = \frac{\tau RD}{180°}$$

Chord Basis





$$10 = R\sin\frac{D}{2}$$

LENGTH OF CURVE

$$L_c = \frac{\pi R I}{180°}$$

$$L_c = \frac{20I}{D} \quad \text{(using arc basis)}$$

TANGENT DISTANCE

$$T = R\tan\frac{I}{2}$$

LENGTH OF LONG CHORD

$$L = 2\,R\sin\frac{I}{2}$$

EXTERNAL DISTANCE

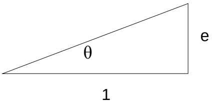
$$E = R\sin\frac{I}{2} - R = R\left(\sec\frac{I}{2} - 1\right)$$

MIDDLE ORDINATE

$$m = R - R\cos\frac{I}{2} = R\left(1 - \cos\frac{I}{2}\right)$$

MINIMUM RADIUS OF CURVATURE

The minimum radius of curve so that a car can round the curve at velocity v without skidding is:



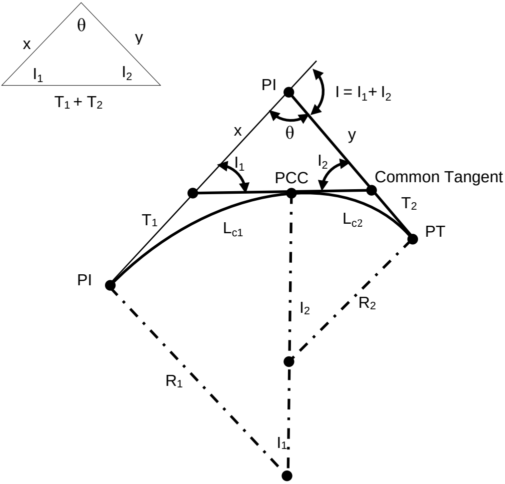
$$\boxed{R_{min} = \frac{v^2}{127\,(e + f)} \quad \text{in meter}}$$

v = design speed in kph
e = superelevation
f = coefficient of friction
Φ= angle of friction

IMPACT FACTOR

$$\boxed{I_f = \tan(\theta + \phi) = \frac{v^2}{gR}}$$

COMPOUND CURVE



PCC = point of compound curve

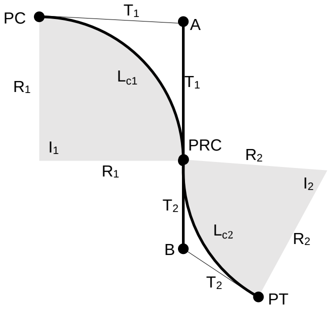
With known stationing of PC:

$$\text{Sta. PT} = \text{Sta. PC} + L_{c1} + L_{c2}$$

With known stationing of PI:

$$\text{Sta. PT} = \text{Sta. PI} - x - T_1 + L_{c1} + L_{c2}$$

REVERSED CURVE



PRC = point of reversed curvature

With known stationing of A:

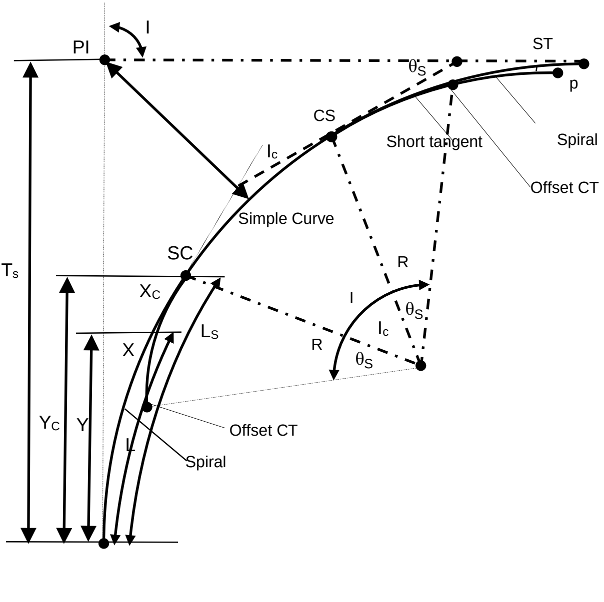
$$\boxed{\text{Sta. PT} = \text{Sta. A} - T_1 + L_{c1} + L_{c2}}$$

With known stationing of PC:

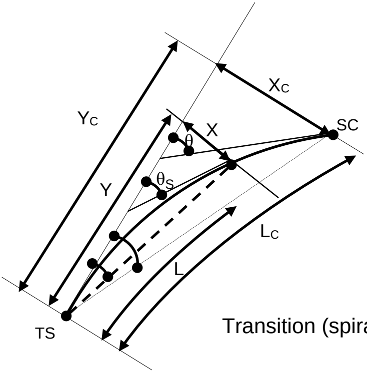
$$\boxed{\text{Sta. B} = \text{Sta. PC} + L_{c1} + L_{c2}}$$

SPIRAL CURVE

A transition curve or spiral curve should be place between tangents and each end of a simple curve and between the simple curves of a compound curve. A spiral increases in curvature gradually, thus avoiding an abrupt change in the rate of lateral displacement of cars. It also provides a means of gradually elevating the far end of the road in proper relation to the degree of curvature.



TS = tangent to spiral
SC = spiral to curve
CS = curve to spiral
ST = spiral to tangent
LT = long tangent
ST = short tangent
R = radius of simple curve



Transition (spiral) Curve

<u>Distance Along Tangent to Any Point on the Spiral</u>

$$Y = L - \frac{L^5}{40R^2 L_s^2} \quad \text{and } Y_c = L_s - \frac{L_s^3}{40R^2}$$

Offset Distance from Tangent to Any Point on the Spiral

$$X = \frac{L^3}{6RL_S} \quad \text{and } X_c = \frac{L_s^2}{6R}$$

Spiral Angle from Tangent to Any Point on the Spiral

$$\theta = \frac{L^2}{2RL_s} \text{ and } \theta_s = \frac{L_s}{2R} \text{, radians}$$

Deflection Angle from TS to Any Point on the Spiral

$$i = \frac{\theta}{3} \text{ and } i_s = \frac{\theta_s}{3}$$

$$\text{and } \frac{i}{i_s} = \frac{L^2}{L_s^2}$$

Tangent Distance

$$T_s = \frac{L_s}{2} + (R + P) \tan\frac{I}{2}$$

Angle of Intersection of Simple Curve

$$I_c = I - 2\theta_s$$

Length of Throw

$$P = \frac{X_c}{4} = \frac{L_s^2}{24R}$$

External Distance

$$E = (R + P)\sec\frac{I}{2} - R$$

Degree of Spiral Curve at Any Point

$$\frac{D}{D_c} = \frac{L}{L_c}$$

D_c = degree of simple curve

Desirable Length of Spiral

$$L_s = \frac{0.036v^2}{R} \quad ;v = \text{velocity in Kph}$$

Super – Elevation Rate e(considering friction)

$$e = \tan\theta$$

$$\tan(\theta + \phi) = \frac{v^2}{gR} \quad ; (v \text{ in } \frac{m}{s} \text{ \& R in m})$$

$$\tan(\theta + \phi) = \frac{0.0079v^2}{gR} \quad ; (v \text{ in kph \& R in m})$$

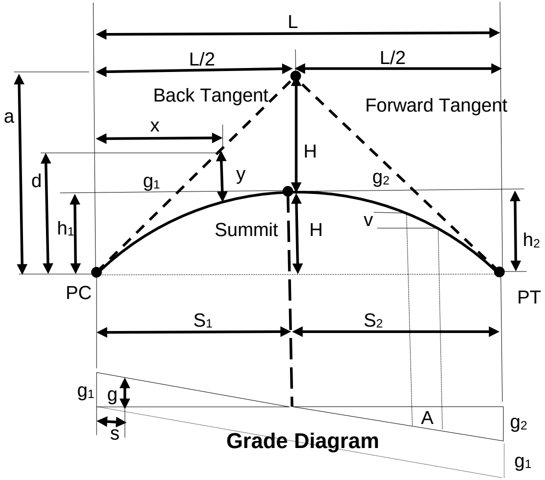
Super – Elevation, e (ideal superelevation)

$$e = \frac{0.0079v^2}{R} \quad ;(v \text{ in kph \& R in meter})$$

Role of Change of Centripetal Acceleration

$$\boxed{q = \frac{v^3}{RL_s}\ ;(m/s^3,\ v \text{ in } \frac{m}{s},R\ \&\ L_s \text{ in m})}$$

PARABOLIC CURVES



Grade Diagram

From the grade diagram shown:

$$\frac{S_1}{g_1} = \frac{L}{g_1 - g_2} \text{ or } S_1 = \frac{g_1 L}{g_1 - g_2}$$

$$\frac{S_2}{g_2} = \frac{L}{g_1 - g_2} \text{ or } S_2 = \frac{g_2 L}{g_1 - g_2}$$

$$v = A$$

$$h_1 = A_{s1} = \frac{1}{2} g_1 S_1$$

$$h_2 = A_{s2} = \frac{1}{2} g_2 S_2$$

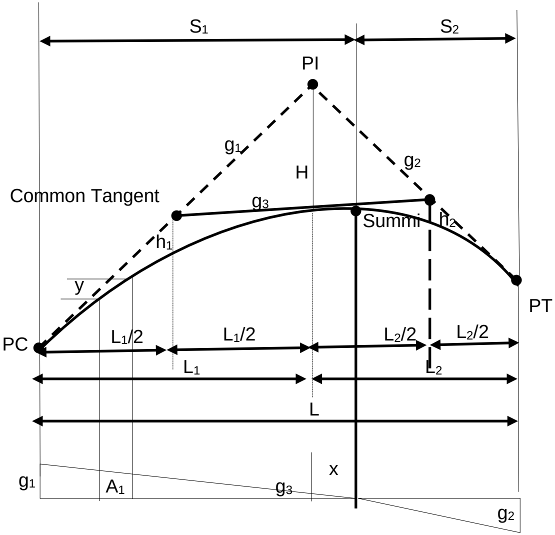
Other Formulas

$$H = \frac{L}{8} \left(g_1 - g_2\right)$$

$$a = (L/2)g_1 \qquad d = g_1 x$$

$$\frac{y}{x^2} = \frac{H}{(L/2)^2}$$

SYMMETRICAL PARABOLIC CURVES



$$H = \frac{(g_1 - g_2)\,L_1L_2}{2L} = \frac{L_1}{2}\,(g_1 - g_3)$$

$$h_1 = h_2 = H/4$$

Slope of Common Tangent

$$g_3 = g_1 - \frac{2H}{L_1} = \frac{g_1L_1 - g_2L_2}{L}$$

Location of highest or lowest point of the curve

When $L_1 g_1 > 2H$, the highest or lowest point is on the right side of the curve

$$\boxed{S_2 = \frac{g_2 L_2^{\,2}}{2H} \text{ and } S_1 = L - S_2}$$

When $L_1 g_1 < 2H$, the highest or lowest point is on the left side of the curve,

$$\boxed{S_1 = \frac{g_1 L_1^{\,2}}{2H} \text{ and } S_2 = L - S_1}$$

The location of the highest or lowest point may also be found using the grade diagram.

SIGHT DISTANCE

Sight distance is the clear visible distance ahead of the driver. This is to categorized as stopping sight distance and passing sight distance.

STOPPING SIGHT DISTANCE (SSD)

This is the length of roadway needed between a vehicle and an arbitrary object (at some point down the road) to permit a driver to stop a vehicle safely before reaching the obstruction.

The minimum SSD is computed for a height of eye(driver eye height) of 3.5 feet and a height of object (obstruction in roadway) of 6 inches.

Stopping sight distance consist of the following time intervals:

1. The time for the driver to perceive the obstruction
2. The time to react
3. The time for the vehicle to stop after brakes are applied (Braking Distance, BD)

The first two time intervals is called perception reaction time. Its ranges from 2 sec to 2.5 sec. It depends on the alertness, care, skill and vision of the driver, and weather.

The distance traveled during the perception – reaction time, t:

$$D = v\,t \text{ , v in meters}$$
$$D = \frac{vt}{3.6} \text{ , v in kph}$$

The third time interval is called the braking distance, BD. It is a function of road inclination and friction. When a car traveling at a velocity v on a road of grade G (positive for upgrade and negative for downgrade) suddenly applies a brake, the Braking Distance is:

$$BD = \frac{v^2}{2g\,(f + G)} \text{ , v in m/s}$$
$$BD = \frac{v^2}{2g\,(f + G)\,(3.6)^2} \text{ , v in kph}$$

Where f is coefficient of friction, G is the road grade (0 if horizontal) and BD is the distance in meters.

The Stopping Sight Distance is therefore:

$$\boxed{\begin{aligned} SSD &= v\,t + \frac{v^2}{2g\,(f+G)}\ ,\ (v \text{ in } \frac{m}{s}\ ,\ t \text{ in sec}) \\ SSD &= v\,t + \frac{v^2}{2g\,(f+G)\,(3.6)^2}\ \ (v \text{ in kph}\ ,\ t \text{ in sec}) \end{aligned}}$$

PASSING SIGHT DISTANCE (PSD)

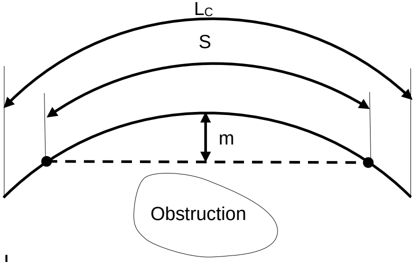
Passing sight distance is the length of roadway ahead visible to the driver.

Minimum Passing Sight Distance for Overtaking Vehicle

This the shortest distance sufficient for a vehicle to turn out of its traffic lane, pass another vehicle, and then turn back to its lane safely without interfering with the overtaken vehicle and the incoming vehicle which was sighted when the overtaking maneuver started

$$\boxed{\text{Minimum PSD} = d_1 + d_2 + d_3 + d_4}$$

SIGHT DISTANCE ON HORIZONTAL CURVES



When $S < L_C$

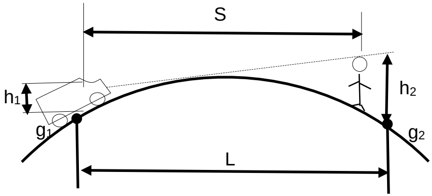
$$R = \frac{S^2}{8m}$$

When $S > L_C$

$$R = \frac{L\,(2S - L)}{8m}$$

Where: L_C = length of curve
R = radius of curve
S = sight distance

SIGHT DISTANCE ON VERTICAL SUMMIT CURVES



When S < L

$$L = \frac{AS^2}{100\left(\sqrt{2h_1}+\sqrt{2h_2}\right)^2}$$

When S > L

$$L = 2S - \frac{200\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}{A}$$

A = change in grade in percent = $g_1 - g_2$

Standard values in road design:

For stopping sight distance (SSD)

h_1 = 3.75 feet (1.14m)

h_2 = 6 inches (0.15m)

For passing sight distance (PSD)

h_1 = 3.75 feet (1.14m)

h_2 = 4.5 inches (1.37 m)

SIGHT DISTANCE ON SAG PARABOLIC CURVES

H = 2 ft (standard design value) $\beta = 1^\circ$

When S < L

$$\boxed{L = \frac{AS^2}{200\,(S\tan\beta + H)}\text{, ft or m}}$$

For H = 2 ft & $\beta = 1^\circ$

$$\boxed{L = \frac{AS^2}{400 + 3.5S}\text{, ft}}$$

When S > L

$$\boxed{L = 2S - \frac{200\,(H + S\tan\beta)}{A}\text{, ft or m}}$$

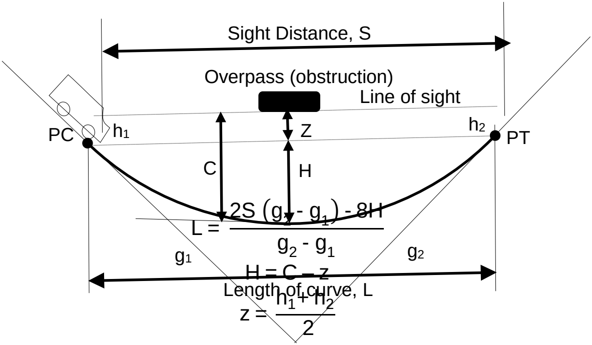
For H = 2 ft & $\beta = 1^\circ$

$$\boxed{L = 2S - \frac{400 + 3.5S}{A}\text{, ft or m}}$$

L = Length of Curve
S = sight distance
A = change in grade in percent = $g_2 - g_1$

SIGHT DISTANCE ON VERTICAL SAG CURVE WITH OBSTRUCTING OVERPASS

When S > L:



$$L = \frac{2S\left(g_2 - g_1\right) - 8H}{g_2 - g_1}$$

$$H = C - z$$

$$z = \frac{h_1 + h_2}{2}$$

When S < L:

$$\boxed{L = \frac{S^2\left(g_2 - g_1\right)}{8H} \qquad H = C - z \qquad z = \frac{h_1 + h_2}{2}}$$

Where C is the vertical clearance between the sag curve and the obstruction (underpass)

PAVEMENTS

Historically, pavements have been divided into two broad categories, rigid and flexible. These classical definitions, in some cases, are an over-simplification. However, the terms rigid and flexible provide a good description of how the pavements react to loads and the environment.

The flexible pavement is an asphalt pavement. It generally consists of a relatively thin wearing surface of asphalt built over a base course and subbase course. Base and subbase courses are usually gravel or stone. These layers rest upon a compacted subgrade (compacted soil). In contrast, rigid pavements are made up of Portland cement concrete and may or may not have a base course between the pavement and subgrade.

The essential difference between the two types of pavements, flexible and rigid, is the manner in which they distribute the load over the subgrade. Rigid pavement, because of concrete's rigidity and stiffness, tends to distribute the load over a relatively wide area of subgrade. The concrete slab itself supplies a major portion of a rigid pavement's structural capacity. Flexible pavement, inherently built with weaker and less stiff material, does not spread loads as well as concrete. Therefore flexible pavements usually require more layers and greater thickness or optimally transmitting load to the subgrade.

One further practical distinction between concrete pavements provides and asphalt pavement is that concrete pavement provides opportunities to reinforce, texture, color and otherwise enhance a pavement, that is not possible with asphalt. These opportunities allow concrete to be made exceedingly strong, long lasting, safe, quiet and architecturally beautiful. Concrete pavements on average outlast asphalt pavements by 10-15 years before needing rehabilitation.

RIGID PAVEMENTS

A rigid pavement typically consist of a Portland cement-concrete slab resting on a subbase course.

Basic Components of Rigid Pavements

Joints

There are three basic joint types used in concrete pavement: contraction, construction and isolation. Specific design requirements for each type depend upon the joint's requirements for each type depend upon the joint's orientation to the direction of the roadway (transverse or longitudinal). Another important factor is load transfer. Except for some isolation joint, all joints provide a means to mechanically connect slabs. The connection helps to spread a load applied on one slab onto slabs along its perimeter(s). This decreases the stress within the concrete and increases the longevity of the joints and slab(s). The efficiency of the mechanical connection is expressed as load transfer efficiency.

Contraction Joints

Contraction joints are necessary to control natural cracking from stresses cause by concrete shrinkage, thermal contraction, and moisture or thermal gradients, within the concrete. Typically transverse contraction joints are cut at a right angle to the pavement centerline and edges. However, some agencies skew transverse contraction joints to decrease dynamic loading across the joints by eliminating the simultaneous crossing of each wheel on a vehicle's axle. Contraction joints are usually sawed into the concrete, but they might be formed or tooled on smaller projects. The details below show the different types of contraction joints and their dimensions.

Construction Joints

Construction joints join concrete that is paved at different times. Transverse construction joints are necessary at the end of a paving segment, or at a placement interruption for a driveway, cross road or bridge. Longitudinal construction joints join lanes that paved at different times, or join through-lanes to curb and gutter or auxiliary lanes. The details below show the different types of construction joints and their dimensions.

Isolation Joints

Isolation joints separate the pavement from objects or structures, and allow independent movement of the pavement, object or structure without any connection that could cause damage, isolation joints are used where a pavement abuts certain manholes, drainage fixtures sidewalks and buildings, and intersects other pavements or bridges. The details below show the different types of isolation joints and their dimensions.

Load transfer

Each type of joint provides a different ability to transfer load across slabs. This ability is termed load transfer efficiency (or effectiveness). It is determined as shown in the figure. Note how both sides of the joint deflect evenly at 100% load transfer efficiency.

Load transfer is important to pavement longevity. Most performance problems with concrete pavement are a result of poorly performing joints. Distress, such as faulting, pumping and corner breaks occur in-part from joints with poor load transfer efficiency. All of these problems worsen when joints deflect greatly under loads.

Dowel bars provide a mechanical connection between slabs without restricting horizontal joint movement. They also keep slabs in horizontal joint movement. When loaded by heavy vehicles, dowel bars lower joint deflection and stress in the concrete slab and reduce the potential problems by increasing load transfer efficiency.
The use of dowel bars (smooth round bars) in transverse contraction joint primarily depends upon the roadway or street classification and can be determined by slab street classification and can be determined by slab thickness. Doweled contraction joints are not usually used in light residential, residential, or secondary urban pavements, but they are used in industrial roads, major streets, highway and airports that will carry heavy

vehicles for long periods. Click here to find out when to use dowels.

When dowels are not used, joint depend solely upon aggregate interlock for load transfer. Aggregate interlock is the mechanical locking which forms between the fractured surfaces along the crack below the joint saw cut. Reliance on aggregate interlock without dowels is acceptable on low-volume and secondary road systems where truck traffic is low and slabs are less than 8 inches thick. Ordinarily, transverse joints with dowel bars provide better load transfer than those relying strictly on aggregate interlock.

Aggregate Interlock Deformed steel tie bars are used in longitudinal joints primarily to prevent lanes from separating. Also by holding slabs tightly together, they promote aggregate interlock and consequently load transfer.

Subbases and Subgrades

A reasonably uniform subgrade or subbase, with no abrupt changes in support, is ideal for any concrete pavement. Most native soils are not too uniform and thus require some improvement or additional layers to compensate.

A subbase is a thin layer of material placed on top of the prepared subgrade. Subbase provide uniform support to the pavement and a stable platform for construction equipments. Subbase also help prevent movement of subgrade soils at transverse pavement joint in roads subject to a large volume of truck traffic. Subbases may be gravel, stone, cement-modified soil, asphalt, or econoconcrete (low-strength concrete)

EMPIRICAL PAVEMENT DESIGN FORMULAS

THICKNESS OF RIGID PAVEMENT

OLDERS THEORY

Without dowels or Tie Bars:

$$\text{Thickness at edge, } t = \sqrt{\frac{3W}{f_{cT}}}$$

With Dowels or Tie Bars

$$\text{Thickness at edge, } t = \sqrt{\frac{3W}{2f_{cT}}}$$

$$\text{Thickness at center, } t = \sqrt{\frac{3W}{4f_{cT}}}$$

Where: W = wheel load in lb or N

f_{cT} = allowable tensile strength of concrete in psi or MPa

t = thickness of concrete slab in inches or mm

AASHTO RIGID PAVEMENT DESIGN EQUATION

$$\log W_{18} = (Z_R)(S_O) + 9.36 \log (SN + 1) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{4.2 - 1.5}\right)}{0.4 + \frac{1094}{(SN + 1)^{5.19}}} + 2.32 \log M_R - 8.07$$

Where: W_{18} = 18,000 lb (80 kN) equivalent single axle loads predicted to p_t.

Z_R = Z-statistic associated with the selected level of design reliability

S_o = overall standard deviation of normal distribution of errors associated with traffic prediction and pavement performance.

SN = Structural Number (essentially a Thickness Index)

ΔPSI = overall serviceability loss = $p_o - p_t$

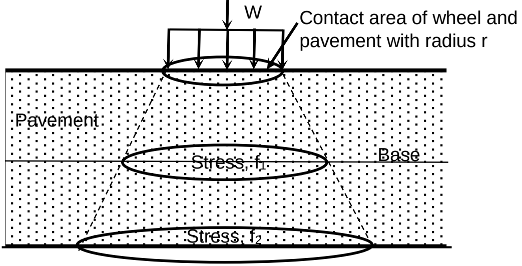
p_o = initial serviceability index following construction

p_t = terminal serviceability index; and

M_R = resilient modulus of the roadbed soil(s)

THICKNESS OF FLEXIBLE PAVEMENT

Cone Pressure Distribution (45˚)



$$t = \sqrt{\frac{W}{\pi f}}$$

where : W= weight load in lb or N

f = bearing strength of subgrade or base in psi or MPa

r = radius of contact of wheel to pavement in inches

Note: To solve t, use f_2 and use f_1 to solve for t_1.

McLeods Method

$$t = K \log \frac{W}{f}$$

Where: W= weight load

f = subgrade pressure

K = constant

Hveem and Carmany (California Highways)

$$K = \frac{F}{0.125}, \text{ where } F = \frac{P}{A}$$

Where: t = thickness of pavement in inches

K = 0.095 (coefficient depending on design wheel load and tire pressure with a factor of safety)

TI = Transfer Index = $1.35\ (EWL)^{0.11}$

R = resistance value

c = cohesiometer value

EWL = equivalent wheel load

Although the above equation encompasses parameters for the bound maters (c value) and the underlying unbound materials (R-value) as well as the traffic volume (TI), it is based on a 5000 lb (22 kN) wheel load with a tire pressure of overtime and the formula shown above was an early form of the procedure.

U.S. Corps of Engineers

$$t = \sqrt{W}\left[\frac{1.75}{CBR} - \frac{1}{\pi p_t}\right]$$

$$CBR = \frac{\text{Unit Load at 0.10 inch penetration}}{\text{1000 psi}} \times 100$$

Where: W = wheel load in kg
CBR = California Bearing Ratio
p_t = tire pressure in kg/cm^2

Stiffness Factor of Pavement

$$\text{Stiffness Factor, } S = \sqrt[3]{\frac{k}{E}}$$

Where: k = modulus of elasticity of subgrade (MPa or psi)
E = modulus of elasticity of pavement (MPa or psi)

Modulus of Subgrade Reaction

$$k = \frac{F}{0.125}\text{, where } F = P/A$$

Where P = load in kg causing 0.125 cm settlement
A = area of standard plate (with 75 cm diameter)

TRAFFIC ENGINEERING

SPACE MEAN SPEED

Space mean speed (harmonic mean speed) u_s is the average speed of vehicles occupying a given length of road at an instant of time

$$\boxed{U_s = \frac{D}{t_{ave}} = \frac{n}{\sum \frac{1}{u_l}} = \frac{nD}{\sum t_l}}$$

Where: D = length of road
n = number of passing vehicle
u_l = velocity of each vehicle

TIME MEAN SPEED OR SPOT SPEED

The arithmetic mean of the speeds of vehicles passing a point during a given interval of time.

$$\boxed{u_t = \frac{1}{n}(u_1+u_2+\ldots+u_n)}$$

FLOW OR FLOW RATE

Flow rate, q is the number of vehicles passing a point during a specified period of time; often referred to as volume when expressed in vehicles per hour (veh/hr) measured over an hour.

$$q = k\, u_s$$

DENSITY

Density, k is the number of vehicles per unit length

SPACING OF VEHICLES

$$\text{Spacing} = \frac{1000}{k} \text{ in meters/veh}$$

$$\text{Spacing} = \frac{u_{ave}}{q}, \text{ (km/veh)}$$

U_{ave} = average speed of passing vehicles in km/hr
q = flow in vehicle/hr

AVERAGE DAILY TRAFFIC

$$\text{ADT} = \frac{\text{No. of passing vehicles per year}}{365}$$

PEAK HOUR FACTOR (PHF)

Peak hour factor is the ration of the traffic flow of the highest volume of traffic in one hour, based on highest five-minute volume of traffic.

$$PHF = \frac{\text{Flow, q } \left(\text{in} \frac{\text{vehicles}}{\text{hour}}\right)}{\text{Highest volume every 5 min x 12}}$$

TRAFFIC INDEX

The traffic index for n year is given as

$$TI = 1.35\ (EWL)^{0.11}$$

$$EWL = \frac{n}{2}\ (1+r)\ (\text{Total annual EWL})$$

$$\text{Total annual EWL = Sum of products of ADT \& EWL}$$

EWL = equivalent wheel load
ADT = Average Daily Traffic
r = rate of increase of traffic, in percent

CAPACITY OF A SINGLE LANE

$$\text{Capacity} = \frac{1000\ v}{s}\ ;\left(\frac{\text{vehicles}}{\text{hr}}\right)$$

$$s = v\,t + L$$

v = average speed of vehicle in kph
s = average center-to-center spacing of vehicles in meters

L = length of one car in meters
t = reaction time in seconds

ACCIDENT RATE

$$\text{Accident Rate} = \frac{\text{No. of Accidents}}{\text{No. of entering vehicles}}$$

Accident rate is usually expressed in accidents per million entering vehicles

Accident Rate per Million Entering Vehicles (MEV) for an intersection

$$R = \frac{N \times 1{,}000{,}000}{ADT\ (t)(365)}$$

Accident Rate per Hundred Million Vehicle Miles of travel (HMVM) for a segment of a highway

$$R = \frac{N \times 1{,}000{,}000}{ADT\ (t)(365)\ (L)}$$

Where : N = number of accidents during the analysis period
ADT = average daily traffic
t = time or period of analysis in years
L = length of segment in miles

FLUID MECHANICS AND HYDRAULICS

Properties of Fluid

Unit Weight or Specific Weight, γ

The weight per unit volume of a fluid

$$\boxed{\gamma = \frac{\text{Weight of Fluid}}{\text{Volume}}}$$

For water $\gamma = 9810\ N/m^3 = 62.4\ lb/ft^3$

Mass Density or Density ρ

The mass of fluid per unit of volume

$$\boxed{\rho = \frac{\text{Mass of Fluid}}{\text{Volume}}}$$

For water, $\rho = 1000\ kg/m^3$

Density of Gases

$$\boxed{\rho = \frac{p}{RT}}$$

Where: p = absolute pressure of gas in kPa
R = gas constant in Joule/kg-˚K
For air, R = 287 J/kg-˚K
T = absolute temperature in degree Kelvin

$$^\circ K = {}^\circ C + 273$$
$$(^\circ \text{Rankine} = {}^\circ \text{Fahrenheit} + 460)$$

Specific Volume, V_s

$$\boxed{V_s = \frac{1}{\rho}}$$

Specific Gravity, s

$$\boxed{s = \frac{\gamma_{fluid}}{\gamma_{water}} = \frac{\rho_{fluid}}{\rho_{water}}}$$

VISCOSITY

The property of a fluid which determines the amount of its resistance to shearing forces. A perfect fluid would have no viscosity.

Dynamic or Absolute Viscosity, μ (mu)

$$\boxed{\mu = \frac{\sigma}{dV/dy} \text{ (Pascal-second or poise)}}$$

Note: 1 poise (P) = 1 dyne-sec/cm^2 = 0.1 Pa-s
1 centiPoise (cP) = 0.001 Pa-s

Kinematic Viscosity, ν (nu)

$$\nu = \frac{\mu}{\rho} \ (m^2/s \ \text{or stroke})$$

Note: 1 stoke (St) = 1 cm^2/s = 0.0001 m^2/s
1 sentiStoke (cSt) = 10^{-6} m^2/s

Surface Tension σ (sigma)

The surface tension of a fluid is the work that must be done to bring enough molecules from inside the liquid to the surface to form a new unit area of that surface in ft-lb/ft^2 or N-m/m^2.

Pressure inside a droplet of liquid

$$p = \frac{4\sigma}{d}$$

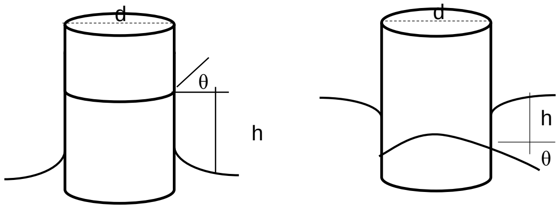
where:

σ = surface tension in N/m
d = diameter of the droplet in meter
p = gage pressure in Pascals

CAPILLARITY

The rise or fall or a fluid in a capillary tube which is caused by surface tension and depends on the relative magnitudes of the cohesion of the liquid and the adhesion of the liquid to the walls of the containing vessel. Liquid rise in tubes they wet (adhesion > cohesion) and fall in tubes they do not wet (cohesion >

adhesion). Capillary is important when using tubes smaller than about 3/8 inch (9.5 mm) in diameter.



Capillary rise Capillary depression

$$\boxed{h = \frac{4\,\sigma \cos\theta}{\gamma\, d}}$$

Use $\theta = 140^\circ$ for mercury on clean glass

For complete wetting, as with water on clean glass, the angle θ is 0°. Hence the formula becomes

$$\boxed{h = \frac{4\,\sigma}{\gamma d}}$$

Where:

h = capillary rise or depression

γ = unit weight
d = diameter of the tube
σ = surface tension

Bulk Modulus of Elasticity, E

The bulk modulus of elasticity of the fluid expresses the compressibility of the fluid. It is the ratio of the change in unit pressure to the corresponding volume change per unit of volume.

$$E = \frac{dp'}{-dv/v} = \frac{\Delta p}{\frac{\Delta v}{v}} \quad (\text{lb/in}^2 \text{ or Pa})$$

Where:

dp' = change in pressure
dv = change in volume
v = volume

COMPRESSION OF GASSES

For a perfect gas:

$$pv^n = p_1 v_1^n = \text{constant}$$

Where p is absolute pressure, v is the specific volume ($v=1/\rho$) and n may have any non-negative value from zero to infinity, depending upon the process to which the gas is subjected. If the process is at constant temperature (isothermal), n = 1.

$$pv = p_1 v_1$$

if there is no heat transfer to and from the gas, the process is known as adiabatic.

$$p_1 v_1^k = p_2 v_2^k$$

A frictionless adiabatic process is called an isentropic process and n is denoted by k, where k = C_p/C_v, the rato of specific heat at constant pressure to that at constant volume.

Boyle's Law (perfect gas)

If the temperature of a given mass of gas remains constant, the absolute pressure of the gas varies inversely with the volume.

$$p = \frac{k}{V} \text{ or } pV = k$$
$$p_1 V_1 = p_2 V_2$$

Charle's or Guy-Lussac's Law (perfect gas)

If a given mass of gas can expand or contract with the pressure remaining constant, the volume V of the gas varies directly as the absolute temperature T, i.e. V/T is constant/

Combined Charle's and Boyle's Law (perfect gas)

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Pressure Disturbances

Pressure disturbances imposed on a fluid move in waves. The velocity or celerity is expressed as:

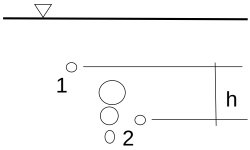
$$\boxed{c = \sqrt{\frac{E_B}{\rho}} \ (m/s \text{ or } ft/s)}$$

where:

c = celerity or velocity of pressure wave in m/s or ft/s
E_B = bulk modulus of elasticity of the fluid in Pa or lb/ft^2

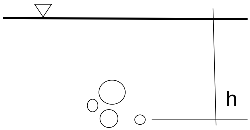
UNIT PRESSURE

Variations in Pressure



The difference in pressure between any two points in a homogeneous fluid at rest is equal to the product of the unit weight of the fluid and the vertical distance between the points

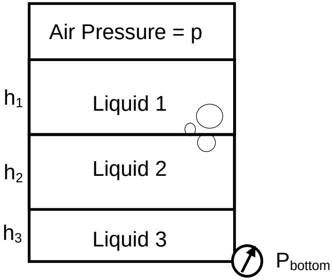
$$\boxed{p_2 - p_2 = \gamma h}$$



The pressure at any point below the free surface of a liquid equals the product of the unit weight of the liquid and the depth of the point.

$$\boxed{p_2 - p_2 = \gamma h}$$

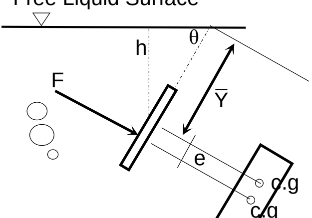
Pressure below layers of different liquids



$$\boxed{P_{bottom} = \sum \gamma\, h + p = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 + p}$$

TOTAL HYDROSTATIC PRESSURE

TOTAL PRESSURE ON PLANE SURFACE

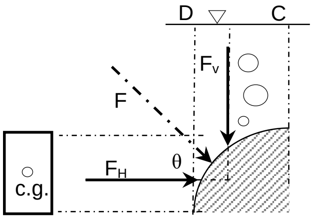


$$F = p_{cg} \times A \qquad \text{or} \qquad F = \gamma \bar{h} A$$

$$e = \frac{I_g}{A\,\bar{Y}} \qquad\qquad \bar{Y} = \frac{\bar{h}}{\sin\theta}$$

where p_{cg} = pressure at the centroid of the plane
I_g = centroidal moment of inertia of the plane
A = area of the plane surface
θ = angle that the plane makes with the horizontal

TOTAL PRESSURE ON CURVED SURFACE



$$F_H = p_{cg} A$$
$$F_v = \gamma V_{ABCD}$$
$$F = \sqrt{F_H^2 + F_v^2}$$
$$\tan\theta = F_v / F_H$$

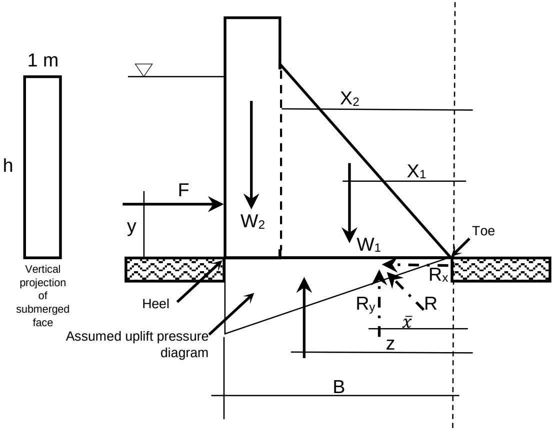
Where: F_H = total force acting on the vertical projection of the curved surface

F_v = the weight of imaginary or real fluid directly above the curved surface

Note: For cylindrical and spherical surfaces, the total force F always passes to the center of the circle defined by its surface

DAMS

Consider 1m length of dam



$$\text{Weight of dam, } W = \gamma_{conc}\ \text{Volume}$$
$$\text{Hydrostatic force, } F = p_{cg}\ A = \gamma\ \bar{h}\ A$$
$$R_y = \sum F_v = W_1 + W_2 - U$$
$$\text{Total uplift pressure, } U = \gamma_w \text{ x vol}_{\text{uplift diagram}}$$

Righting Moment

These are the moments about the toe causing rotation towards the upstream side. From the figure shown,

$$RM = W_1\ x_1 + W_2\ x_2$$

Overturning Moment

These are the moments about the toe causing rotation towards the downstream side. From the figure shown,

$$OM = Fy + U\ z$$

Factor of Safety against Sliding

$$FS_s = \frac{\mu\ R_y}{R_x}$$

Where μ is the coefficient of friction between the foundation and base of dam

Factor of Safety against Overturning

$$FS_o = \frac{RM}{OM}$$

Location of R

$$R_y\,\bar{x} = RM - OM$$

FOUNDATION PRESSURE (SOIL PRESSURE)

Eccentricity, e

$$e = \frac{B}{2} - \bar{x}$$

For e ≤ B/6:

$$q = -\frac{R_y}{B}\left(1 \pm \frac{6e}{B}\right)$$

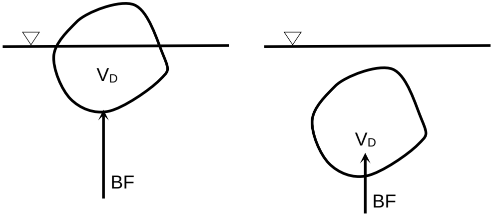
Use (+) for the pressure at the toe
Use (–) for the pressure at the heel

For e ≥ B/6:

$$q = \frac{2R_y}{3\,\bar{x}}$$

BOUYANCY

Archimedes' Principle – Any body immersed in a fluid is acted upon by an unbalanced upward force called the buoyant force, which is equal to the weight of the fluid displaced.

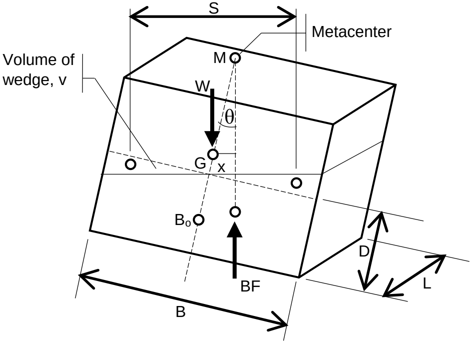


$$BF = \gamma_F V_D$$

For homogeneous body floating on a homogenous liquid. The volume displace is:

$$V_D = \frac{\gamma_{body}}{\gamma_{liquid}} V_{body} = \frac{S_{body}}{S_{liquid}} V_{body}$$

STATICALLY STABILITY OF FLOATING BODY



$$MB_o = \frac{vS}{V_D \sin\theta}$$

$$RM \text{ or } OM = W\,(x) = W\,(MG \sin\theta)$$

Where v = volume of the wedge of immersion

s = horizontal distance between the centroid of the wedges

V_D = volume displaced

θ = angle of tilting

If the body has the shaped of a rectangular parallelepiped

$$MB_o = \frac{B^2}{12D}\left(1 + \frac{\tan^2\theta}{2}\right)$$

Where B = width, D = draft

Metacentric Height

Metacentric height is the distance from the metacenter to the center of gravity of the body measure along the axis of the body.

$$\boxed{MG = MB_o \pm GB_o}$$

Value of MB_o in the Upright Position

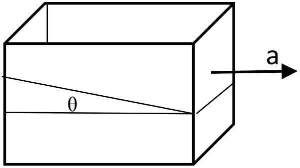
(Initial Value)

$$\boxed{MB_o = \frac{1}{V_D}}$$

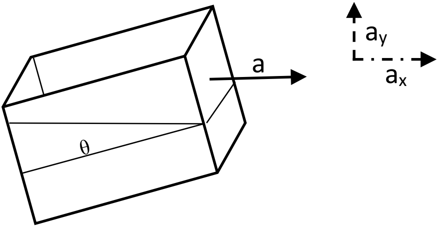
Where I = moment of inertia of the body along the waterline section

RELATIVE EQUILIBRIUM OF LIQUIDS

Horizontal Acceleration



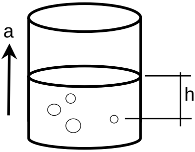
$$\boxed{\tan\theta = \frac{a}{g}}$$



$$\tan\theta = \frac{a_h}{g \pm a_v}$$
$$a_h = a\cos\alpha \quad ; \quad a_v = a\sin\alpha$$

Use (+) if the motion is upward and (-) if downwards.

Vertical Acceleration



$$p = \gamma\, h \left(1 \pm \frac{a}{g}\right)$$

Use (+) for upward motion and (-) for downward motion.

ROTATION

$$y = \frac{\omega^2 x^2}{2g}$$

$$\tan\theta = \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

Volume of Paraboloid

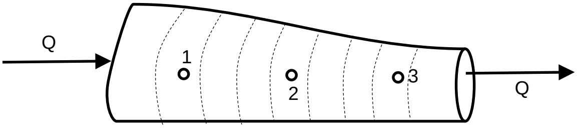
$$\text{Volume} = \frac{1}{2}\pi r^2 h$$

FLUID FLOW AND PIPES

Flow Rate

Volume Flow Rate, $Q = Av$
Mass Flow Rate, $M = \rho Q$
Weight Flow Rate, $W = \gamma Q$

Continuity Equation



Incompressible fluid

$$Q_1 = Q_2 = Q_3 = \ldots$$
$$A_1v_1 = A_2v_2 = A_3v_3 = \ldots$$

Compressible fluid

$$\rho_1 Q_1 = \rho_2 Q_2 = \rho_3 Q_3 = \ldots$$

Where A = cross-sectional area of flow
v = mean velocity of flow

Reynold's Number (for pipes)

Reynolds Number R is the ratio of inertia forces to viscous forces

$$R = \frac{vD_\rho}{\mu} = \frac{vD}{v}$$

Where v = mean velocity of flow, m/s
D = pipe diameter, m
μ = (mu) dynamic viscosity (Pa-s)
v = (nu) kinematic viscosity (m^2/s) = μ/ρ
ρ = density, kg/m^3

For non-circular pipes, use D = 4R, where R is the hydraulic radius, R = A/P

For R < 2100, the flow is laminar

Laminar flow in circular pipes can be maintained up to values or R as high as 50,000. However, n such cases this type of flow in inherently unstable and the least disturbance will transform it instantly into turbulent flow. On the other hand, it is practically impossible for turbulent flow in a straight pipe to persist at values of R much below 2100 because any turbulence that is set up will be damped out by viscous friction.

ENERGY EQUATION

Total Energy of Flow

E = kinetic Energy + Potential Energy

$$E = \frac{v^2}{2g} + \frac{\rho}{\gamma} + Z$$

Where

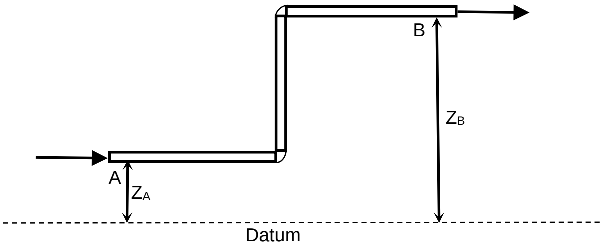
$\frac{v^2}{2g}$ = velocity head (K.E.)

$\frac{\rho}{\gamma}$ = pressure head (P.E.)

Z = elevation head (P.E.)

Bernoulli's Energy Theorem

Between any two points (1 and 2) along the stream:



$$E_1 + HA - HE - HL = E_2$$

Where: E_1 = Total Energy (head) at section 1
HA = head added (by the pump)
HE = head extracted (by turbine or any other device)
HL = total head lost

HEAD LOST IN THE PIPE FLOW

MAJOR HEAD LOST (FRICTIONAL LOSSES)

DARCY-WEISBACH FORMULA

$$h_f = \frac{fL}{D}\,\frac{v^2}{2g} \text{ in ft or meter}$$

For Laminar Flow

$$f = \frac{64}{R} = \frac{64\,\mu}{v\,D_\rho}$$

$$h_f = \frac{32\,\mu\,L\,v}{\rho\,g\,D^2}$$

For non-circular pipe, use D = 4R

For circular pipes, the following formulas may be used

$$\frac{v^2}{2g} = \frac{8\,Q^2}{\pi^2 g\,D^4} \qquad h_f = \frac{fL}{D}\,\frac{8\,Q^2}{\pi^2 g\,D^4}$$

$$\text{For S.I. units, } h_f = \frac{0.0826\, f\, L\, Q^2}{D^5}$$

$$h_f = \frac{128\, \mu\, L\, Q^2}{\pi\, \rho\, g\, D^4} \text{ (for laminar flow)}$$

MANNING'S FORMULA

$$\text{S.I. units, } v = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \text{ , (m/s)}$$

$$\text{English unit, } v = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \text{ ,(ft/s)}$$

Where R = hydraulic radius = A/P
S = slope of EGL = h_f / L

$$h_f = \frac{6.35\, n^2 L\, v^2}{D^{4/3}} \text{ (m)}$$

Use D = 4R for non-circular pipes

For circular pipes, the following formula may be used

$$h_f = \frac{10.29\, n^2\, L\, Q^2}{D^{\frac{16}{3}}} \text{ (m)}$$

HAZEN-WILLIAMS FORMULA

English Units

$$v = 1.318\ C_1 R^{0.63} S^{0.54}\ ,\ v \text{ in } \frac{ft}{s}\ ,\ R \text{ in ft}$$

For circular pipes, this formula becomes

$$Q = 0.4322\ C_1\ D^{2.63}\ S^{0.54},\ Q \text{ in } \frac{ft^3}{s}\ ,\ D \text{ in ft}$$

S.I. Units

$$v = 0.849\ C_1 R^{0.63} S^{0.54}\ ,\ v \text{ in } \frac{m}{s}\ ,\ R \text{ in m}$$

For circular pipes, this formula becomes

$$Q = 0.2785\ C_1\ D^{2.63}\ S^{0.54},\ Q \text{ in } \frac{ft^3}{s}\ ,\ D \text{ in ft}$$

$$\text{and } h_f = \frac{10.67\ L\ Q^{1.85}}{C_1^{\ 1.85}\ D^{4.87}}$$

Where:

R = hydraulic radius

S = slope of EGL = h_f / L

C_1 = Hazen William's coefficient

MINOR HEAD LOST

Minor losses are due to changes in direction and velocity of flow, and is expressed in terms of the velocity head at the smaller section of the pipe in case of constrictions

$$\boxed{h_m = K\frac{v^2}{2g}}$$

Where K = coefficient of minor loss

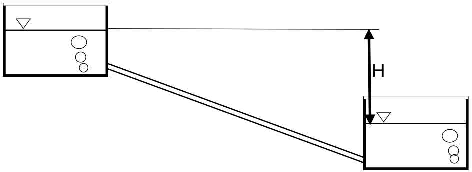
HEAD LOST THROUGH NOZZLES

$$\boxed{h_n = \left(\frac{1}{C_v^{\ 2}}\right)\frac{v_n^{\ 2}}{2g}}$$

For horizontal pipes with uniform diameter, the head lost between any two points is equal to the difference in pressure head between the points.

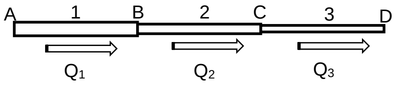
$$\boxed{HL = \frac{P_2 - P_1}{\gamma}}$$

For a pipe or system of pipes connecting two reservoirs, the total head lost is equal to the difference in water surface elevation of the reservoirs.



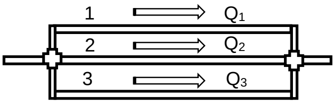
$$HL = H$$

PIPIE IN SERIES



$$Q_1 = Q_2 = Q_3$$
$$HL = h_{L1} + h_{L2} + h_{L3}$$

PIPE IN PARALLEL



$$Q = Q_1 + Q_2 + Q_3$$
$$HL = h_{L1} = h_{L2} = h_{L3}$$

EQUIVALENT PIPE

For a pipe or system of pipes (O), the equivalent single pipe (E) is must satisfy the following conditions:

$$Q_E = Q_O$$
$$\text{and } HL_E = HL_O$$

ORIFICE AND TUBES

The velocity and discharge through an orifice is given by

$$v = C_v \sqrt{2gH}$$
$$Q = C\,A_o \sqrt{2gH}$$
$$C = C_c \times C_v$$

Where C_v = coefficient of velocity
C = coefficient of discharge
C_c = coefficient of contraction
H = total head in meter or feet of the fluid flowing

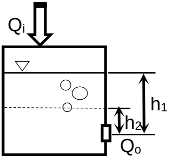
Value of H

$$H = \text{head upstream} - \text{head downstream}$$
$$H = h_u + \frac{v_a^2}{2g} + \frac{p_u}{\gamma} - h_o - \frac{p_o}{\gamma}$$

Where

v_a = velocity approach
p_u = pressure at the upstream side
p_o = pressure at the downstream side

UNSTEADY FLOW (VARIABLE HEAD)



If water flows into a tank at the rate of Q_i and at the same time leaves Q_o, the time for the level to change from h_1 to h_2 is

$$\boxed{t = \int_{h_1}^{h_2} \frac{A_s\,dh}{Q_i - Q_o}}$$

If $Q_i = 0$ (no inflow)

$$\boxed{t = \int_{h_1}^{h_2} \frac{A_s\,dh}{Q_o}}$$

If the outflow is through an orifice under a variable head H

$$\boxed{Q_o = C\,A_o\sqrt{2gH}}$$

If the cross-sectional area A_s is constant and the flow is through an orifice, the formula becomes

$$\boxed{t = \frac{2\,A_s}{C\,A_o\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)}$$

Where H_1 = initial head (at level 1)

H_2 = final head (at level 2)

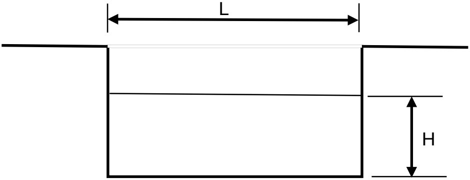
If water flows through the pipe connecting the two tanks shown, the time for the head to change from H_1 to H_2 is

$$t = \frac{A_{s1}A_{s2}}{A_{s1}+A_{s2}} \; \frac{2\ A_s}{C\ A_o\sqrt{2g}} \left(\sqrt{H_1} - \sqrt{H_2}\right)$$

WEIR

Weirs are overflow structure usually built across an open channel to control or measure the flow

RECTANGULAR WEIR (SUPPRESSED)



General Formula

$$Q = \frac{2}{3}\ C\ \sqrt{2g}\ L\left[(H + h_v)^{2/3} - h_v^{\ 2/3}\right]$$
$$\text{or } Q = C_w\ L\left[(H + h_v)^{2/3} - h_v^{\ 2/3}\right]$$

where $h_v = \frac{v_a^{\ 2}}{2g}$ velocity head of approach

C = coefficient of discharge
C_w = weir factor

Neglecting v_a:

$$Q = \frac{2}{3}\, C \sqrt{2g}\, L\, H^{\frac{2}{3}}$$

$$\text{or } Q = C_w\, L\, H^{\frac{2}{3}}$$

FRANCIS FORMULA (C_W = 1.84 FOR S.I. UNITS)

$$Q = 1.84\, L\left[(H + h_v)^{2/3} - h_v^{\,2/3}\right]$$

Neglecting v_a:

$$Q = 1.84\, L H^{\frac{2}{3}}$$

Using English Units C_w = 3.33

Contracted Weirs

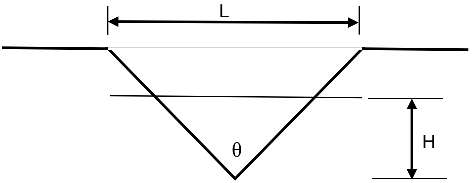
For contracted weirs, the value of L is reduced by 10% of the head H in each end contraction

For one end contraction, use L = L – 0.10H
For two one end contraction, use L = L – 0.20H

CIPOLLETI WEIR

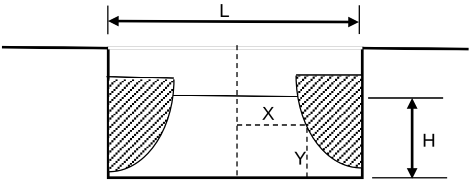
$$Q = 1.859\ L\ H^{\frac{2}{3}}$$
$$\theta = 75.9637° = 75°\ 57'\ 50''$$
$$\beta = 14.0363° = 14°\ 2'\ 10''$$

TRIANGULAR V-NOTCH WEIR



$$Q = \frac{8}{15}\ C\ \sqrt{2g}\ \tan\frac{\theta}{2}\ H^{\frac{5}{2}}$$

SUTTRO WEIR (PROPORTIONAL FLOW WEIR)



$$Q = C\,\pi\,K\sqrt{2g}\,H$$
$$K = x\sqrt{y}$$

UNSTEADY FLOW WEIR (VARIABLE HEAD)

$$t = \int_{H_1}^{H_2} \frac{A_s\,dh}{Q_o}$$

If the flow is through a suppressed rectangular weir

$$t = \frac{2\,A_s}{C_w\,L}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

where C_w = weir factor, H_1 = initial head, H_2 = final head

OPEN CHANNEL

Open channels are conduits which fluid flows with a free surface in an open channel flow, the hydraulic grade line is coincident with the stream surface and the flow may be uniform or non-uniform.

Chezy Formula

The mean velocity of flow in an open channel may be computed by the Chezy Formula

$$\boxed{v = C\sqrt{RS}}$$

where C = Chezy coefficient
R = hydraulic radius
S = slope of the EGL

$$\boxed{R = \frac{\text{Cross-sectional area of flow}}{\text{Wetted Perimeter}}}$$

Value of C

1. Manning (S.I.)

$$\boxed{C = \frac{1}{n} R^{\frac{1}{6}}}$$

2. Kutter (S.I.)

$$\boxed{C = \frac{\frac{1}{n} + 23 + \frac{0.00155}{S}}{1 + \frac{n}{\sqrt{R}}\left(23 + \frac{0.00155}{S}\right)}}$$

Where n is the roughness coefficient

MANNING'S FORMULA

SI Units

$$\boxed{\begin{array}{c} v = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \text{, (m/s)} \\ Q = A\,\frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \text{, (m/s)} \end{array}}$$

English Units

$$v = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \text{, (ft /s)}$$
$$Q = A \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \text{, (ft/s)}$$

UNIFORM FLOW

The simplest of all open channel problem is the uniform flow condition. In uniform flow, the slope of the energy grade line (EGL) is equal to the slope of the channel bed, S_o

$$S = S_o$$

Head Lost under Uniform Flow

$$HL = S_o \text{ x } L$$

L = length of channel

Normal Depth

The normal depth d_n is the depth at which uniform flow will occur in an open channel. Normal depth may be determined from the following equation for discharge:

$$\text{Chezy : } Q = AC\sqrt{RS}$$
$$\text{Mannings: } Q = A \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

SPECIFIC ENERGY, H

The specific energy (H) is defined as the energy per unit weight relative to the bottom of the channel. It is given by:

$$\boxed{H = \frac{v^2}{2g} + d}$$

MOST EFFICIENT SECTIONS

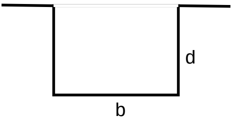
Also known as most economical sections, these are sections which for a given slope, area, and roughness, the rate of discharge is maximum. In order to attain maximum efficiency, the perimeter of the canal must be minimized.

PROPORTIONS FOR MOST EFFICIENT SECTIONS

Rectangular Section

$b = 2d$

$R = d/2$

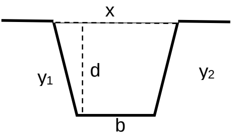


Trapezoidal Section

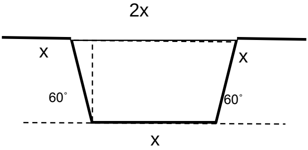
Top width = sum of sides

$x = y_1 + y_2$

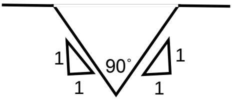
$R = d/2$



Note: The most efficient of all trapezoidal section is the half regular hexagon



Triangular Section
90˚ V-notched



The most efficient of all section is the Semi-Circle

For Circular Section

Discharge, Q, is maximum when depth = 0.938 D
Velocity, v, is maximum when depth = 0.82 D

<u>FROUDE NUMBER</u>

The ratio of the inertia force to gravity force and is given by the expression:

$$\boxed{F = \frac{V}{\sqrt{gL}}}$$

For rectangular channel L = depth of flow d.

$$\boxed{F = \frac{V}{\sqrt{gd}}}$$

ALTERNATE STAGE OF FLOW

For a given total specific energy H, for an open channel flow, there exist two stages or depths of flow that will give the same discharge. These are the upper stage and the lower stage.

Upper Stage

Flow is tranquil
Depth is called subcritical depth
Froude Number, $F < 1$

Lower Stage

Flow is rapid or shooting
Depth is called supercritical depth
Froude Number, $F > 1$

Critical Depth, d_c

Critical depth is the depth at which for a given total specific energy H, the discharge is maximum or it is the depth at which for a given discharge Q, the total specific energy is minimum.

Critical depth is characterized by:

1. Critical Velocity
2. Critical Slope

3. Froude Number, $F = 1$

Critical depth is obtained by differentiating the following equation with respect to d and setting $dQ/dd = 0$

$$\boxed{Q = A\sqrt{2g\,(H-d)}}$$

Where A is the cross-sectional area of flow (which is a function of d and H is the specific energy which is constant.

Critical Depth for Rectangular Section

The critical depth for a rectangular canal can be obtained by the formula,

$$\boxed{d_c = \sqrt[3]{\frac{q^2}{g}} = \frac{2}{3}\ H}$$

Where q is the unit discharge (m^3/s per meter width)

$$\boxed{q = v\ d = \frac{Q}{b}}$$

The slope required to give uniform flow at critical depth is known as the critical slope S_c. The equation for critical slope for wide rectangular channel is:

$$\boxed{S_c = \frac{g\,n^2}{d_c^{\frac{1}{3}}}}$$

Critical Depth for Any Section

The critical depth for any section may be computed from the formula

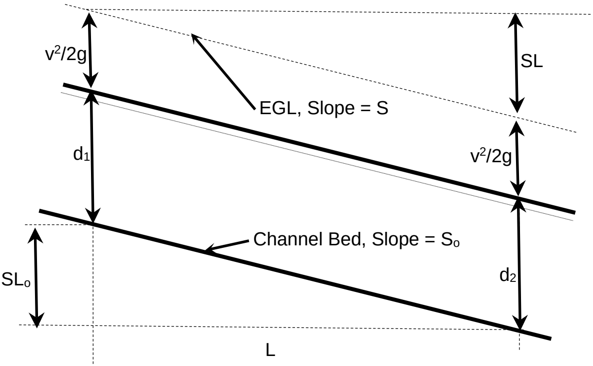
$$\boxed{\frac{Q^2}{g} = \frac{A^3}{B}}$$

Where A is the cross-sectional area of flow and B is the flow width at the top

Note: A and B are always in terms of d, except that for rectangular canal B is constant

NON-UNIFORM OR VARIED FLOW ($S \neq S_o$)

Uniform flow rarely occurs in natural streams because of changes in depth width, and slope along the channel. The Manning's equation for uniform flow can be applied to non-uniform with an accuracy dependent on the length of reach L taken. Thus a long stream can be divided into several reaches of varying length such that the change in depth is roughly the same within each reach.



With reference to the figure above, the length of reach L is

$$L = \frac{\left(\frac{v_2{}^2}{2g} + d_2\right)\left(\frac{v_1{}^2}{2g} + d_1\right)}{S_o - S} = \frac{H_2 - H_1}{S_o - S}$$

The value of S can be computed using Manning Formula

$$v_m = \frac{1}{n} R_m{}^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$S = \left(\frac{n\, v_m}{R_m{}^{\frac{2}{3}}}\right)^2$$

Where

v_m = mean velocity between the two section 1 and 2

$$v_m = \frac{v_1 + v_2}{2}$$

R_m = mean hydraulic radius = $\frac{R_1 + R_2}{2}$

HYDRAULIC JUMP

A hydraulic jump occurs when the upstream flow is supercritical (F>1). To have a jump there must be a flow impediment downstream. The downstream impediment could be a weir, a bridge abutment, a dam or simply a channel friction. Water depth increases during a hydraulic jump and energy is dissipated as turbulence. Often engineers will purposely install impediments in channels in order to force humps to occur. Mixing of coagulant chemicals in water treatment plants is often aided by hydraulic jumps. Concrete blocks may be installed in a channel downstream of a spillway in order to force a jump to occur thereby reducing the velocity and energy of the water. Flow will go from supercritical (F>1) to subcritical (F<1) over a jump

General Equation

$$\boxed{A_2 \overline{h_2} - A_1 \overline{h_1} = \frac{Q^2}{g}\left(\frac{1}{A_1} - \frac{1}{A_2}\right)}$$

For Rectangular Canal

$$\boxed{\frac{q^2}{g} = \frac{d_1 d_2 (d_1 + d_2)}{2}}$$

$$\text{Length, } L = 220\, d_1 \tanh \frac{F_1 - 1}{22}, \text{ in meter}$$

Where q= v d = Q/b

F_1 = Froude number at section 1

Head Lost in the Jump

$$HL = \left(\frac{v_1^2}{2g} + d_1\right) - \left(\frac{v_2^2}{2g} + d_2\right)$$

For rectangular canal, the head lost may be computed by:

$$HL = \frac{(d_2 - d_1\ )}{4\, d_1\, d_2}$$

HYDRODYNAMICS

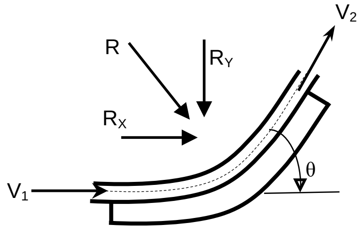
Hydrodynamics deals with the study of the motion of a fluid and of the interactions of the fluid with its boundaries. The force developed by this moving fluid is called the dynamic force.

Force against Fixed Flat Plate Held Normal to the Jet

If a jet of water strikes a fixed flat plate held normal (perpendicular) to its path, the dynamic force developed is given by the formula

$$\text{Dynamic force, } F = \frac{Q_\gamma}{g} v = \rho\, Q\, v$$

Force against Fixed Curved Vane

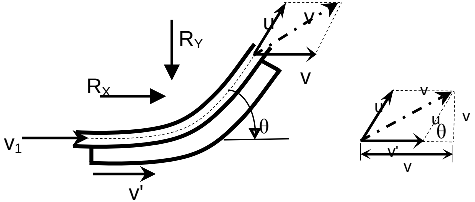


$$F_x = \frac{Q_\gamma}{g}\,(v_{1x} - v_{2x})\ ;\ F_y = \frac{Q_\gamma}{g}\left(v_{1y} - v_{2y}\right)$$

$$F = \sqrt{F_x^2 + F_y^2}$$

Where: v_1 = velocity of the jet before hitting the vane
v_2 = velocity of the jet as it leaves the vane

Force Against a Moving Vane



$$F_x = \frac{Q'_\gamma}{g}(v_{1x} - v_{2x}) \; ; \; F_y = \frac{Q'_\gamma}{g}\left(v_{1y} - v_{2y}\right)$$

$$Q' = A\,u \qquad u = v_{1y} - v_{2y}$$

u = relative velocity of the jet as it moves along the vane
Q’ = amount of fluid deflected by the vane

If the vane is frictionless, such that the jet leaves the vane with relative velocity (u) in the direction of θ:

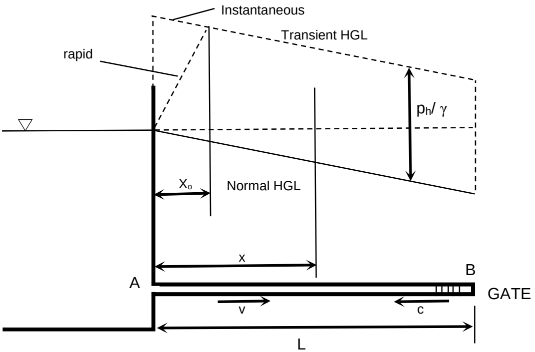
$$F_x = \rho A\,(v-v')^2\,(1-\cos\theta)$$

$$F_y = -\rho A\,(v-v')^2\,(1-\sin\theta)$$

Where v’ = velocity of the vane
u = relative velocity of the jet
Q’ = quantity of water deflected by the vane
v_1 = absolute velocity of the jet before it strikes the vane
v_2 = absolute velocity of the jet as it leaves the vane

WATER HAMMER

Water hammer is the term used to express the resulting shock (hammer rise) in a pipeline cause by the sudden decrease or stoppage of motion (rate of flow or velocity) of the fluid



Consider the pipe line shown leading from a reservoir A to the valve at B. If the value is suddenly closed, the lamina of the liquid next to the valve will be compressed by the rest of the column of liquid flowing against it. At the same time the walls of the pipe surrounding this lamina will be stretched by the excess pressure produce. The cessation of flow and resulting pressure increase move along the pipe as a wave with the velocity c which is given by the following equations:

For rigid pipes:

$$\boxed{c = \sqrt{\frac{E_B}{\rho}}}$$

For non-rigid pipes

$$\boxed{c = \sqrt{\frac{E_B}{\rho\left(\frac{E_B d}{Et}\right)}}}$$

The time for the pressure wane to travel from A to B and back again is:

$$\boxed{T = \frac{2L}{c}}$$

Instantaneous Closure ($t_c = 0$)

The resulting shock due to instantaneous closure is given by:

$$\boxed{p_h = \rho\, cV}$$

For instantaneous closure, the pressure increase reaches up to the pipe entrance at A where it drops instantly to the value it would have for zero flow.

Rapid Closure ($t_c < 2L/c$)

It is physically impossible for a valve to be closed instantaneously ($t_c = 0$). For a rapid closure ($t_c < 2L/c$) the maximum pressure near the valve would still be

$$\boxed{p_h = \rho\, cV}$$

No matter how rapid the valve closure may be, so long as it is not the idealized instantaneous case, there will be some distance x_o from the intake within which the pressure rise cannot extend all the way to the reservoir intake:

Slow Closure (t_c > 2L/c)

For slow closure, the excess pressure produced decreases uniformly from the value at the valve to zero at the intake. The maximum water-hammer pressure p_h developed is given approximately by:

$$\boxed{p_h = \frac{2L\,\rho\,v}{t_c}}$$

Where:

- c = celerity of pressure wave in m/s
- E_B = bulk modulus of elasticity of the fluid in Pa
  - (for water at 30 ˚C, $E_B = 2.25 \times 10^6$ Pa)
- E = modulus of elasticity of the pipe wall in Pa
- t = pipe thickness in mm
- d = internal diameter of pipe in mm
- t_c = time of closure in seconds
- L = length of pipe in m
- v = velocity of flow in m/s
- p_h = pressure change due to water hammer in Pa
- ρ = density of the fluid in kg/m^3

GEOTECHNICAL ENGINEERING

PROPERTIES OF SOIL

Density and Unit Weight of Water

Density of water, ρ_w = 1000 kg/m^3

ρ_w = 1 kg/liter = 1 gram/cc

Unit weight of water, γ_w = 9.81 kN/m^3

Basic Formulas

Unit weight of substance, $\gamma_s = G\,\gamma_w$

Weight of water, $W_w = \gamma_w\, V_w$

Weight of substance, $W_s = \gamma_s\, V_s = G\,\gamma_w\, V_s$

Specific gravity of substance, $G_{subs} = \gamma_{subs}/\gamma_w$

Physical Properties of Soil

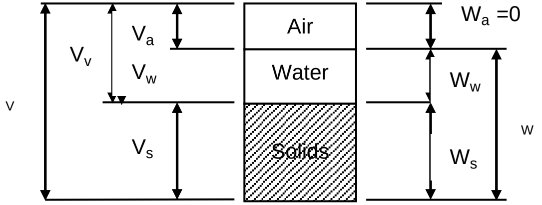


Figure Face Diagram of Soil

The following relationships can be made from the phase diagram shown:

$$\text{Total weight of soil, } W = W_w + W_s$$
$$\text{Volume of voids, } V_v = V_s + V_w$$
$$\text{Total volume, } V = V_s + V_v$$

Void Ratio, e

Void ratio is the ratio between the volumes of voids to the volume of solids of a soil mass. It is usually expressed in percent.

$$e = \frac{V_v}{V_s}$$

Note: $0 < e < \infty$

Porosity, n

Porosity is the ratio between the volumes of voids to the total volume of a mass. It is usually expressed in percent.

$$n = \frac{V_v}{V}$$

Note: $0 < n < 1$

Relationship between e and n

$$n = \frac{e}{1 + e} \quad \text{and} \quad e = \frac{n}{1 - n}$$

Water Content or Moisture Content, MC or w

The ratio of the weight of water to the weight of solid particles.

$$MC \text{ or } w = \frac{W_w}{W_s} \times 100\%$$

Note: $0 < MC < \infty$

Degree of saturation, S

The ratio of the volume of water to the volume of voids

$$S = \frac{V_w}{V_v} \times 100$$

Degree of saturation varies from S = 0 for completely dry soil and S = 100% for totally saturated soil.

Relationship between G, MC, S and e

$$G \times MC = S \times e$$

Unit Weight (or bulk unit weight) of Soil Mass, γ_m

$$\gamma_m = \frac{W}{V}$$

$$\gamma_m = \frac{G + Se}{1 + e}\,\gamma_w = \frac{G + GMC}{1 + e}\,\gamma_w$$

Dry Unit Weight, γ_d

For dry soils, S = 0 and MC = 0

$$\gamma_d = \frac{W_s}{V} = \frac{G}{1 + e}\,\gamma_w$$

$$W_s = \frac{\gamma_d}{1 + MC}$$

$$\gamma_d = \frac{\gamma_m}{1 + MC}$$

Saturated Unit Weight, γ_{sat}

For saturated soils, S =1, $V_v = V_w$

$$\gamma_{sat} = \frac{G + e}{1 + e}\,\gamma_w$$

Submerged or Buoyant Unit Weight, γ_b or γ'

$$\gamma_b \text{ or } \gamma' = \gamma_{sat} - \gamma_w$$

$$\gamma_b \text{ or } \gamma' = \frac{G - 1}{1 + e}\,\gamma_w$$

Critical Hydraulic Gradient

Critical hydraulic gradient is the hydraulic gradient that brings a soil mass (essentially, coarse-grained soils) to static liquefaction (quick condition).

$$i_{cr} = \frac{\gamma_b}{\gamma_w} = \frac{G - 1}{1 + e}$$

OTHER FORMULAS

These formulas may not be memorized. These can be derived from the previous formulas.

$$\text{Volume of voids, } V_v = \frac{e}{1 + e}\,V$$

$$\text{Volume of solid, } V_s = \frac{V}{1 + e}$$

$$\text{Volume of water, } V_w = \frac{Se}{1+e} V$$

$$\text{Weight of water, } W_w = \frac{Se}{1+e} V \gamma_w$$

$$\text{Weight of solid, } W_s = \frac{1}{1+e} V G_m \gamma_w$$

$$\text{Weight of soil, } W = \frac{G+Se}{1+e} V \gamma_w$$

$$\text{Dry unit weight, } \gamma_d = \frac{\gamma_m}{1+MC}$$

Specific Gravity of Some Minerals

Mineral	Specific Gravity
Gypsum Volcanic Ash	2.32
Orthoclase	2.56
Kaolinite	2.61
Quartz	2.67
Calcite	2.72
Dolomite	2.87
Magnetite	5.17

RELATIVE DENSITY OF GRANULAR SOILS

The relative density, Dr expresses the state of compactness of a natural granular soil.

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

$$\text{or } D_r = \frac{{}^{1}/_{\gamma_{min}} - {}^{1}/_{\gamma_{d}}}{{}^{1}/_{\gamma_{min}} - {}^{1}/_{\gamma_{max}}}$$

Where:

e_{max} = void ratio of the soil in the loosest state.
e_{min} = void ratio of the soil in densest state.
e = void ratio of the soil deposit (in-situ sate)
$\gamma_{d\ max}$ = dry unit weight in densest state.
$\gamma_{d\ min}$ = dry unit weight in loosest state.
γ_d = dry unit weight in-situ state.

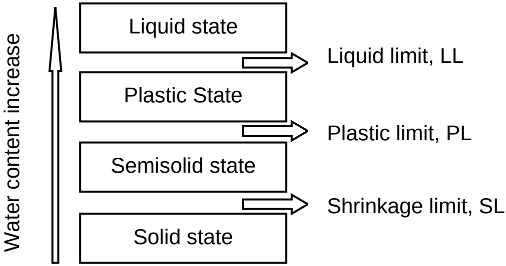
Designation of Granular Soils

Designation	D_r(%)
Very Loose	0 - 15
Loose	15 - 53
Medium Dense	35 - 70
Dense	70 - 85
Very Dense	85 - 100

<u>CONSISTENCY</u>

Consistency is the term used to describe the degree of firmness (e.g., soft, medium, firm or hard) of a soil.

The consistency of a cohesive soil is greatly affected by the water content of the soil. A gradual increase of the water content of the soil may transform a dry soil from solid state to a semisolid state, to a plastic state and after further moisture increase, into a liquid state. The water content at the corresponding junction points of these states are known as the shrinkage limit, the plastic limit and the liquid limit respectively.



Soil Indices

Index	Definition	Correlation
Plasticity	PI =LL-PL	Strength, compressibility, compactibility, …
Liquidity	$LI = \frac{MC-PL}{PI}$	Compressibility and stress rate
Shrinkage	SI =PL -SL	Shrinkage potential
Activity of clay	$A_c = \frac{PI}{\mu}$	Swell potential and so forth

where μ =percent of soil finer than 0.002 mm (clay size)

Activity	Classification
$A_c < 0.7$	Inactive clay
$0.7 < A_c < 1.2$	Normal clay
$A_c = 1.2$	Active clay

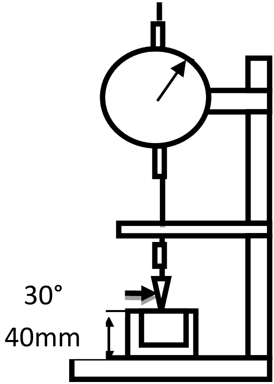
Description of Soil based on Liquidity Index

LI < 0	Semisolid state – high strength, brittle (sudden) fracture is expected
0 <LI < 1	Plastic state – intermediate strength, soil deforms like a plastic material
LI >1	Liquid state – low strength, soil deforms like a viscous fluid

Description of Soil based on plasticity index

PI	Description
0	Nonplastic
1 - 5	Slightly plastic
5 - 10	Low plasticity
10 - 20	Medium plasticity
20 - 40	High plasticity
> 40	Very high plasticty

Fall Cone Method to Determine Liquid and Plastic limits



Fall cone apparatus

Fall cone test (cone penetration test) offers more accurate method of determining both the liquid limit and plastic limit. In this test a cone with apex angle of 30° and total mass of 80 grams is suspended above, but just in contact with, the soil sample. The cone is permitted to fall freely for a period of 5 seconds. The water content corresponding to a cone penetration of 20 mm defines the liquid limit.

The liquid limit is difficult to achieve in just a single test. In this regard, four or more test at different moisture content is required. The results are plotted as water content (ordinate, arithmetic scale) versus penetration (abscissa, logarithmic scale) and the best-fit straight line (liquid state line) linking the data points is drawn (see figure below) the liquid limit is read from the plot as the water content on the liquid state line corresponding to penetration of 20 mm.

The plastic limit is found by repeating the test with a cone of similar geometry, but with a mass of (M_2) 240 grams. The liquid state line for this cone will be bellow the liquid state line for the 80-gram cone (M_1) and parallel to it.

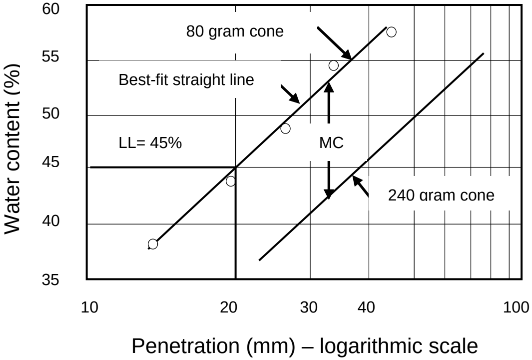


Figure – Typical fall cone result

The plastic limit is given as:

$$\boxed{PL = LL - \frac{2\Delta MC}{\log \frac{M_2}{M_1}}}$$

Cup Method to Determine Liquid Limit

The device used in this method consists of a brass cup and a hard rubber. The brass cu is dropped onto the base by a cam operated by a crank.

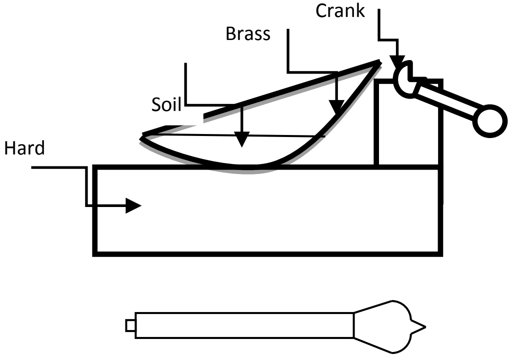


Figure Liquid limit device and grooving tool

The soil paste is placed in the cup, a groove is then cut at the center of the soil pat with the standard grooving tool. By the use of the crank-operated cam, the cup is lifted and dropped from a height of 10 mm. The moisture content required to close a distance of 12.7 mm along the bottom of the groove after 25 blows is defined as the liquid limit.

Since it is difficult to adjust the moisture content to meet the required closure after 25 blows, at least three tests for the same soil are conducted at varying moisture contents, with the number of blows required to achieve closure varying between 15 and 35. The results are plotted on a graph paper, with the moisture content along the vertical axis (algebraic scale) and the number of blows, N, along the horizontal axis (logarithmic scale). The graph is approximated as a straight line (called the flow curve). The moisture content corresponding to N = 25 is the liquid limit of the soil. The slope of the flow line is defined as the flow index and may be written as:

$$\boxed{\text{Flow index, FI} = \frac{MC_1 - MC_2}{\log\left({}^{N_2}\!/\!_{N_1}\right)}}$$

Where MC_1 and MC_2 are the moisture contents, in percent corresponding to the number of blows N_1 and N_2, respectively

N	15	20	22	30	36
MC	48	45.5	44.7	43.5	42.3

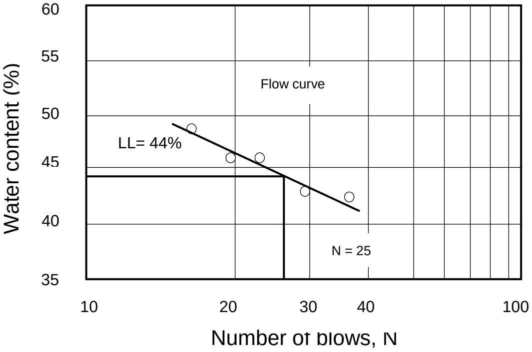


Figure Flow Curve

One-Point method to determine Liquid Limit

This method may be used when only one test is run for the soil. This is established by the U.S. corps of Engineers in 1949 and was also adopted by ASTM under designation D-4318.

$$\boxed{LL = MC_N \left(\frac{N}{25}\right)^{\tan\beta}}$$

Where:
N = number of blows in the liquid limit device for a 0.5-in groove closure.
MC_N =corresponding moisture content
tan β =0.121 (but note that tan β is not equal to 0.121 for all soils)
This method yields good results for the number of blows between 20 and 30

Shrinkage Limit

Soil shrinks as moisture is gradually lost from it. With continuing lost of moisture, a stage of equilibrium is reached at which more loss of moisture will result in no further volume change. The moisture content, in percent, at which the volume of the soil mass ceases to change, is defined as the shrinkage limit.

The shrinkage limit is determined as follows. A mass of wet soil, m_1, is placed in a porcelain dish 44.5 mm in diameter and 12.5 mm high and then oven-dried. The volume of the oven-dried soil is determined by using mercury to occupy the vacant spaces caused by shrinkage. The mass of mercury is determined and the volume decrease caused by shrinkage can be calculated from the known density of mercury. The shrinkage limit is calculated from

$$SL = \frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2}\,\rho_w$$

Where:

m_1 = mass of wet (saturated)soil
m_2 = mass of oven-dried soil
V_1 = volume of wet soil
V_2 = volume of oven-dried soil
ρ_w = density of water

Shrinkage Ratio

$$SR = \frac{1}{\rho_w}\,\frac{m_2}{V_2}$$

Specific Gravity of Solids

$$G = \frac{1}{\dfrac{1}{SR} - \dfrac{SL}{100}}$$

Liquidity Index and Consistency Index

Liquidity index (LI) defines the relative consistency of a cohesive soil in the natural state

$$\text{Liquidity index, LI} = \frac{MC - PL}{LL - PL}$$

Where MC = in situ or natural moisture content if MC is greater than LL, LI > 1. If MC < PL, LI < 0

$$\text{Consistency Index, CI} = \frac{LL - MC}{LL - PL}$$

If MC is equal to LL, CI is zero. If MC = PI, CI = 1

Atterberge's limits are also used to assess the potential swell of a given soil

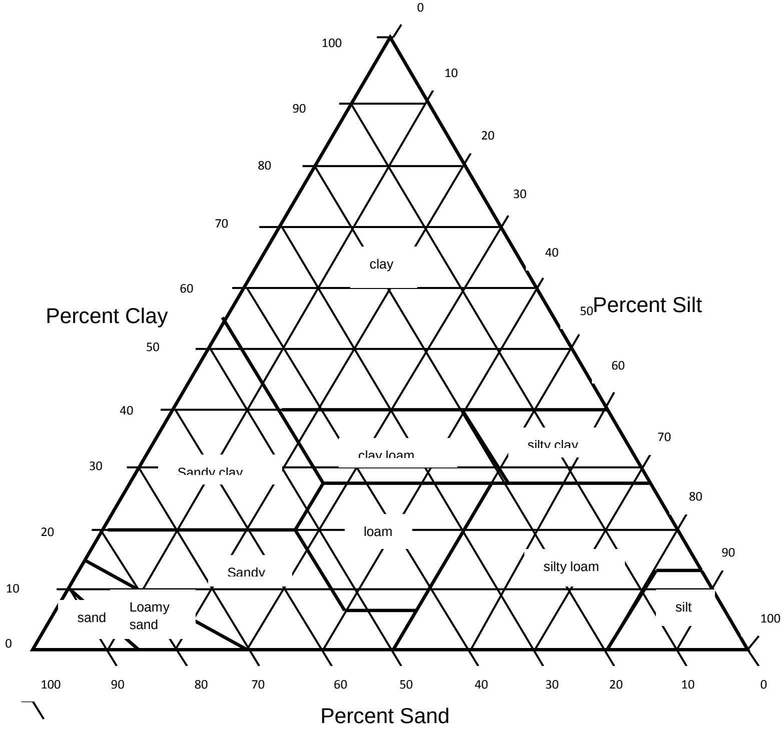
LL	PI	Potential swell classification
< 50	< 25	Low
50 – 60	25 – 35	Medium
> 60	> 35	High

CLASSIFICATION OF SOIL

TEXTURAL CLASSIFICATION

In this classification system, the soils are named after their principal components, such as sandy clay, silty clay, silty loam, and so on. There are number of classification system developed by different organizations. Shown below is the one developed by the U.S. Department of Agriculture (USDA). This method is based on the following limits of particle size:

Sand size : 2.0 to 0.05 mm in diameter
Silt size : 0.05 to 0.002 mm in diameter
Clay size : smaller than 0.002 mm in diameter



UNIFIED SOIL

CLASSIFICATION SYSTEM

This system classifies soils into two broad categories.

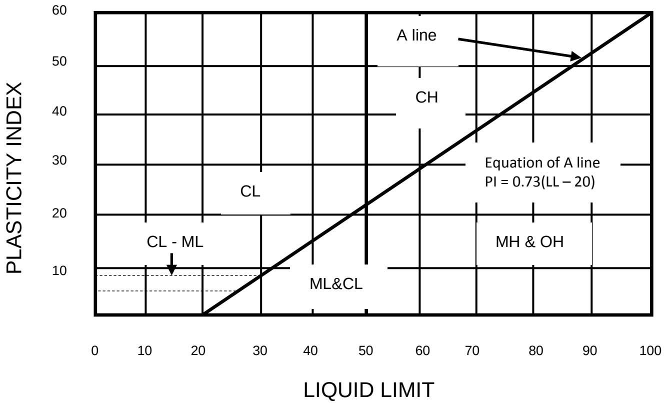
1. Coarse-grained soil that are gravelly and sandy in nature with less than 50% passing through the No. 200 Sieve, the group symbols start with prefixes of either G or S. G for gravel or gravelly soil, and S for sand or sandy soil.
2. Fine-grained soil with 50% or more is passing through the No. 200 sieve. The group symbol start with prefixes of M, which stands for inorganic silt, C for inorganic clay, and O for inorganic clay. The symbol Pt is used for peat, muck, and other highly organic soils.

UNIFIED SOIL CLASSIFICATION SYSTEM (USCS)

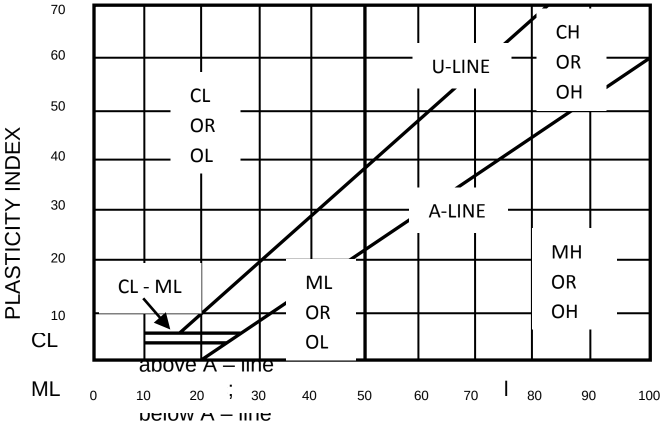
<table>
<tr><th colspan="3">Major Divisions</th><th></th><th>Typical Names</th><th></th><th colspan="2">Classification Criteria</th></tr>
<tr><td rowspan="8">Coarse-Grained Soils: More than 50% retained on No. 200 sieve</td><td rowspan="4">Gravels 50% or more of coarse fraction retained on No. 4 sieve</td><td rowspan="2">Clean Gravels</td><td>GW</td><td>Well-graded gravels and gravel-sand mixtures, little or no fines</td><td rowspan="8">Classification on basis of percentage of fines
Less than 5% pass No. 200 sieve – GW,GP, SW, SP
More than 12% pass No. 200 sieve – GM, GC, SM, SC
5% to 12% pass No. 200 sieve – borderline classification requiring use of dual symbols</td><td colspan="2">$Cu = (D_{60}/D_{10}) > 4$
$Cc = (D_{30})^2/(D_{10} x D_{60})$
Between 1 and 3</td></tr>
<tr><td>GP</td><td>Poorly-graded gravels and gravel-sand mixtures, little or no fines</td><td colspan="2">Not meeting both criteria for GW</td></tr>
<tr><td rowspan="2">Gravel with fines</td><td>GM</td><td>Silty gravels, gravel-sand-silt mixtures</td><td>Atterberg limits plot below “A” line or plasticity index less than 4</td><td rowspan="2">Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols</td></tr>
<tr><td>GC</td><td>Clayey gravels, gravel-sand-clay mixtures</td><td>Atterberg limits plot above “A” line or plasticity index greater than 7</td></tr>
<tr><td rowspan="4">Sands more than 50% of coarse fraction passes No. 4 sieve</td><td rowspan="2">Clean Sand</td><td>SW</td><td>Well graded sands and gravely sands, little or no fines</td><td colspan="2">$Cu = (D_{60}/D_{10}) > 6$
$Cc = (D_{30})^2/(D_{10} x D_{60})$
Between 1 and 3</td></tr>
<tr><td>SP</td><td>Poorly graded sands and gravely sands little or no fines</td><td colspan="2">Not meeting both criteria for SW</td></tr>
<tr><td rowspan="2">Sand with fines</td><td>SM</td><td>Silty sands, sand-silt mixtures</td><td>Atterberg limits plot below “A” line or plasticity index less than 4</td><td rowspan="2">Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols</td></tr>
<tr><td>SC</td><td>Clayey sands, sand-clay mixtures</td><td>Atterberg limits plot above “A” line or plasticity index greater than 7</td></tr>
</table>

<table>
<tr><td rowspan="6">Coarse-Grained Soils: More than 50% retained on No. 200 sieve</td><td rowspan="3">Silts and Clays Liquid Limit 50% or less</td><td>ML</td><td>Inorganic silts very fine sands, rock flour, silty or clayey fine sands</td><td rowspan="6">SEE GRAPH BELOW</td></tr>
<tr><td>CL</td><td>Inorganic clays of low to medium plasticity, gravely clays, sandy clays, silty clays, lean clays</td></tr>
<tr><td>OL</td><td>Organic silts and organic silty clays of low plasticitry</td></tr>
<tr><td rowspan="3">Silts and Clays Liquid Limit greater than 50%</td><td>MH</td><td>Inorganic silts micaceous or diatomaceous fine sands or silts, elastic silts</td></tr>
<tr><td>CH</td><td>Inorganic clays of high plasticity, fat clays</td></tr>
<tr><td>OH</td><td>Organic clays of medium to high plasticity</td></tr>
<tr><td colspan="2">Highly Organic Soils</td><td>PT</td><td>Peat, muck, and other highly organic soils</td><td>Visual-Manual identification, See ASTM Designation D2488</td></tr>
</table>

Other symbols used
W – well graded
P – poorly graded
L – low plasticity (LL < 50)
H – high plasticity (LL > 50)



PLASTICITY INDEX
60
50
40
30
20
10
A line
CH
Equation of A line
PI = 0.73(LL – 20)
CL
CL - ML
MH & OH
ML&CL
0
10
20
30
40
50
60
70
80
90
100
LIQUID LIMIT



PLASTICITY INDEX
70
60
50
40
30
20
10
CH
OR
OH
U-LINE
CL
OR
OL
A-LINE
MH
OR
OH
CL - ML
ML
OR
OL
CL
above A – line
ML
below A – line
0
10
20
30
40
50
60
70
80
90
100

OL - Organic; (LL – oven-dried)/(LL – not dried) < 0.75; LL < 50
CH - Inorganic; LL ≥ 50; Atterberg limits plot on or Above A – line
MH - Inorganic; LL ≥ 50; Atterberg limits plot below A – line
OH - Organic; (LL – oven-dried)/(LL – not dried) < 0.75; LL is greater than or equal to 50
CL – ML - Inorganic; Atterberg limits plot in the hatched zone

Particle-Size Distribution Curve (Sieve Analysis)

Sieve analysis consists of shaking the soil sample through a set of sieves that have the smaller openings. These sieves are generally 200 mm in diameter.

To conduct a sieve analysis, the soil is first oven-dried and then all lumps must be broken into smaller particles. The soil is then shaken through a stack of sieves with openings of decreasing size from top to bottom. A pan is placed below the stack.

A particle-size distribution curve can be used to determine the following four parameters for a given soil:

Effective Size, D_{10}

This parameter is the diameter in the curve corresponding to 10% finer. The effective size of a granular soil is a good measure to estimate the hydraulic conductivity and drainage through soil.

Uniformity Coefficient, C_u

$$C_u = \frac{D_{60}}{D_{10}}$$

Where D_{60} = diameter corresponding to 60%

Coefficient of Gradation or Coefficient of Curvature, C_c

$$C_c = \frac{(D_{10})^2}{D_{60} \times D_{10}}$$

Where D_{30} = diameter corresponding to 30% finer

Sorting Coefficient, S_o

$$S_o = \sqrt{\frac{D_{75}}{D_{25}}}$$

Where
D_{75} = diameter corresponding to 75% finer
D_{25} = diameter corresponding to 25% finer

AASHTO CLASSIFICATION SYSTEM

According to this system, soil is classified into seven major groups: A-1 through A-7. Soils classified under groups A-1, A-2 and A-3 are granular materials of which 35% or less of the particles pass through the No. 200 sieve. Soils of which more than 35% pass through the No. 200 sieve are classified under groups A-4, A-5, A-6 and A-7. These soils are mostly silt and clay-type materials.

To classify the soil using the tables below, one must apply the test data from left to right. By process of elimination, the first group from the left into which the test data fit is the correct classification.

To evaluate the quality of a soil as a highway subgrade material, one must also incorporate a number called the group index with the groups and subgroups of the soil. This index is written in parentheses after group of subgroup designation, example, A-7-5(35).

$$\boxed{GI = (F_{200} - 35)[0.2 + 0.005\ (LL - 40)] + 0.01(F_{200} - 15)(PI - 10)}$$

Where:
F_{200} = percentage passing No. 200 sieve
LL = liquid limit, PI = plasticity index

The first in the GI formula is the partial group index determined from liquid limit. The second term is the partial group index determined from plasticity index.

- If GI yields a negative value, it is taken as 0
- GI is rounded-off to the nearest whole number.
- There is no upper limit for GI
- The GI of soils belonging to groups A-1-a, A-1-b, A-2-4, A-2-5, and A-3 is always zero.
- When calculating the GI for soils that belong to groups A-2-6 and A-2-7, use the partial GI for PI, or

$$GI_P = 0.01(F_{200} - 15)(PI - 10)$$

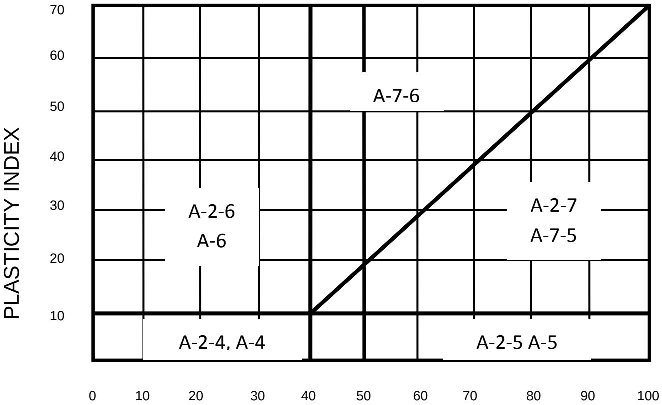
Classification of Highway Subgrade Materials for Granular Materials (AASHTO)

General classification	Granular materials (35% or less of total sample passing No. 200)						
Group classification	A-1			A-2			
	A-1-a	A-1-b	A-3	A-2-4	A-2-5	A-2-6	A-2-7
Sieve analysis (percentage passing)							
No. 10	50max.						
No. 40	30max.	50max.	51min.				
No. 200	15max	25max.	10max.	35max.	35max.	35max.	35max.
Characteristic of fraction passing No.40							
Liquid limit				40max.	41min.	40max.	41min.
Plasticity index	6max.		NP	10max.	10max.	11min.	11min.
Usual types of significant constituent materials	Stone fragment, gravel, and sand		Fine sand	Silty or clayey gravel and sand			
General subgrade rating	Excellent to good						

Classification of Highway Subgrade Materials for Silt-Clay Materials (AASHTO)

General classification	Silt-Clay materials (more than 35% of total sample passing No. 200)			
Group classification	A-4	A-5	A-6	A-7 A-7-5[a] A-7-6[b]
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36min.	36min.	36min.	36min.
Characteristic of fraction passing No. 40				
Liquid limit	40max.	41min.	40max.	41min.
Plasticity index	10max.	10max.	11min.	11min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			

[a] For A-7-5, PI ≤LL-30
[b] For A-7-6, PI >LL-30



FLOW OF WATER THROUGH SOILS

DARCY'S LAW

Darcy's law governs the flow of water through soils. Darcy (1856) proposed that the average flow velocity through soils is proportional to the gradient of the total head. The velocity of the flow is:

$$v = k\,i$$

$$\text{Seepage velocity, } v_s = {}^{v}/_{n}$$

Where

$i = \frac{H}{L}$ =hydraulic gradient

k = coefficient of permeability or hydraulic conductivity, m/s or m/day

n = porosity

The flow of water is:

$$Q = k\,i\,A$$

DETERMINATION OF THE COEFFICIENT OF PERMEABILITY

Constant-Head Test

The constant-head test is used to determine the coefficient of permeability of coarse-grained soils.

$$\boxed{k = \frac{V\,L}{t\,A\,h}}$$

Where:

- V = volume of water collected in time t
- h = constant head
- A = cross-sectional area of the soil
- L = length of soil sample
- T = duration of water collection

Falling Head Test

The falling head test is used for fine-grained soils because of the flow of water through these soils is too slow to get reasonable measurement from the constant head test

$$\boxed{k = \frac{aL}{A(t_2 - t_1)} \ln\left(\frac{h_1}{h_2}\right)}$$

Where:

- a = cross-sectional area of the standpipe
- h_1 = head at time t_1
- h_2 = head at time t_2

Effect of Water temperature on k

The hydraulic conductivity of soil is a function of unit weight of water, and thus, it is affected by water temperature. The relationship is given by:

$$\boxed{\frac{k_{T_1}}{k_{T_2}} = \frac{\mu_{T_2}}{\mu_{T_1}} \frac{\gamma_{wT_1}}{\gamma_{wT_2}}}$$

Where
k_{T_1}, k_{T_2} =hydraulic conductivities at temperatures T_1 and T_2, respectively
μ_{T_1}, μ_{T_2} =viscosity of water at temperatures T_1 and T_2, respectively
γ_{wT_1},
γ_{wT_2} = unit weight of water at temperatures T_1 and T_2, respectively

Flow Through Permeable Layers

hydraulic gradient is:

$$\text{Hydraulic gradient, } i = \frac{h}{\frac{L}{\cos\alpha}}$$

$$\text{hydraulic gradient, } i = \sin\alpha$$

EMPIRICAL RELATIONS FOR HYDRAULIC CONDUCTIVITY

Hazen Formula (for fairly uniform sand):

$$\boxed{k(\text{cm / sec}) = c(D_{10})^2}$$

Where:
c = a constant that varies from 1 to 1.5
D_{10} = effective size, mm

Casagrande (for fine to medium clean sand):

$$\boxed{k = 1.4\, e^2\, k_{0.85}}$$

Where k = hydraulic conductivity at a void ratio e
$k_{0.85}$ = k at void ratio of 0.85

Kozeny-Carman Equation

$$\boxed{k = C_1 \frac{e^3}{1 + e}}$$

Where k is the hydraulic conductivity at a void ratio of e and C, is a constant.

Samarasinhe, Huang, and Drnevich

$$\boxed{k = C_3 \frac{e^n}{1 + e}}$$

Where C_3 and n are constants to be determined experimentally.

EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOIL

The equivalent permeability in the x-direction is (parallel flow):

$$(k_x)_{eq}\, H = \sum k_x z$$

$$(k_x)_{eq}\, H = k_{x1} z_1 + k_{x2} z_2 + \ldots k_{xn} z_n$$

The equivalent permeability in the z-direction is (normal flow):

$$\frac{H}{(k_z)_{eq}} = \sum \frac{z}{k_z}$$

$$\frac{H}{(k_z)_{eq}} = \frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}$$

FLOW THROUGH LAYERS OF AQUIFERS

$$k_{eq}(H) = k_1H_1 + k_2H_2$$

Flow per unit width, $q = k_{eq}\ i\ a$

$$i = \frac{D_1 - D_2}{L};\ a = \frac{D_1 + D_2}{2}(1)$$

$$\frac{L}{k_{eq}} = \frac{L_1}{k_1} + \frac{L_2}{k_2}$$

Flow per unit width, $q = k_{eq}\ i\ a$

$$i = \frac{D_1 - D_2}{L};\ a = \frac{D_1 + D_2}{2}(1)$$

HYDRAULIC OF WELLS

Underground water constitutes an important source of water supply. The stratum of soil in which this water is present is known as an aquifer. On the basis of their hydraulic characteristics, well are divided into two categories: gravity or water table wells, and artesian or pressure wells. If the pressure at the surface of the surrounding underground water is atmospheric, the well is of the gravity type; if this pressure is above atmospheric because an impervious soil stratum overlies the aquifer, the well is artesian.

Assume that the water surrounding a well has a horizontal surface under static conditions. The lateral flow of water toward the well requires the existence of a hydraulic gradient, this gradient being caused by a difference in pressure. To create this difference in pressure, the surface of the surrounding water assumes the shape of an inverted “cone” during pumping of the well, as shown in profile in the figure. This cone is known as the cone of depression, the cross section of the cone at the water surface is called the circle of influence, and the distance through which the water surface is lowered at the well is termed the drawdown of 1 m is called specific capacity of the well.

GRAVITY WELL

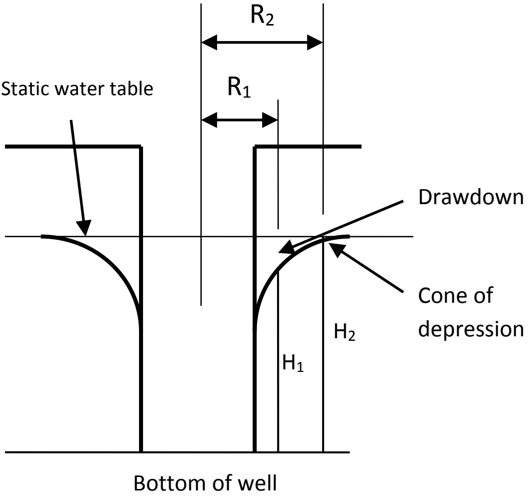


Figure: Gravity well

$$Q = \frac{\pi k\left(h_2^{\ 2} - h_1^{\ 2}\right)}{\ln(R_2/R_1)}$$

Artesian Well

Artesian wells are wells drilled through impermeable rocks into strata where water is under enough pressure to force it to the surface without pumping

$$Q = \frac{2\pi kt(\ h_2 - h_1\ )}{\ln(R_2/R_1)}$$

Where:

h_1, h_2, R_1, R_2 are in meters

k = coefficient of permeability

Q = discharge in m^3 / hr

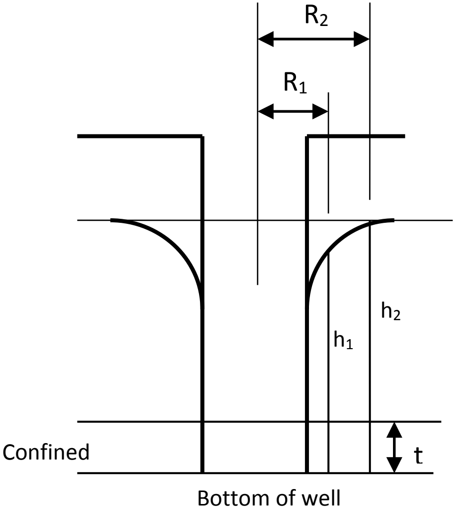


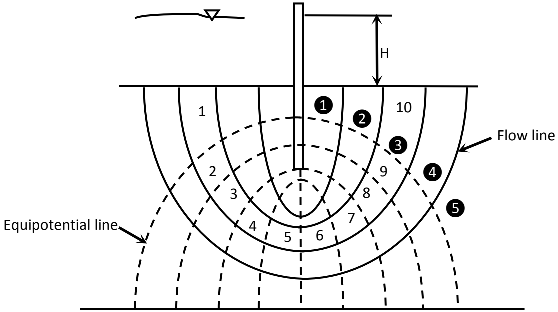
Figure: Artesian

TWO-DIMENSIONAL FLOW OF WATER THROUGH SOILS

Flow Nets

Seepage losses through the ground or through earth dams and levees and the related flow pattern and rate of energy loss, or dissipation of hydrostatic head, are frequently estimated by means of a graphical technique known as flow net.

Flow net is a graphical representation of a flow field that satisfies Laplace's equation and comprises a family of flow lines and equipotential lines.



Flow nets

A flow net must meet the following criteria

1. The boundary conditions must be satisfied
2. Flow lines must intersect equipotential lines at right angles,

3. The area between flow lines and equipotential lines must be curvilinear squares. A curvilinear square has the property that an inscribed circle can drawn to touch each side of the square and continuous bisection results, in the limit, in a point.
4. The quantity of flow through each flow channel is constant.
5. The head loss between each consecutive equipotential line is constant.
6. A flow line cannot intersect another flow line.
7. An equipotential line cannot intersect another equipotential line

Flow line is the path followed by a particle of water as it moves through a saturated soil mass.

Equipotential line is a line connecting points of equal potential energy

The flow of water through isotropic soil is:

$$\boxed{q = kH\frac{N_f}{N_d}}$$

Where:

K = coefficient of permeability

H = head

N_f = number of flow channels

= number of flow lines minus 1

N_d = number of equipotential (pressure) drops

= number of equipotential line minus one

N_f/N_d is called the shape factor

If the soil is anisotropic

$$q = H \frac{N_f}{N_d} \sqrt{k_x k_z}$$

STRESSES IN SOIL

Intergranular Stress, p_E (Effective stress)

Intergranular or efective stress is the stress resulting from particle-to-particle contact of soil.

$$p_E = p_T - p_w$$

Pore Water Pressure, p_w (Neutral stress)

Pore water pressure or neutral stress is the stress induced by water-pressures.

$$p_w = \gamma_w h_w$$

Note: For soils above water table, $p_w = 0$.

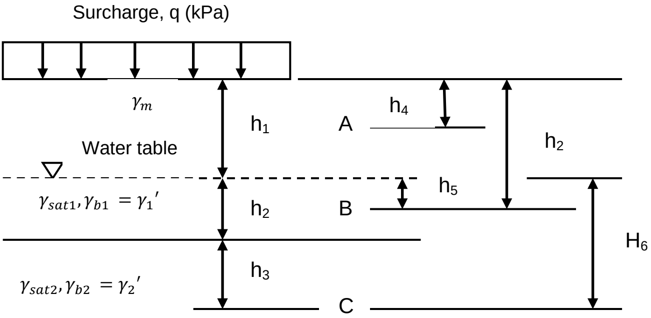
Total Stress, p_T

The sum of the effective and neutral stresses.

$$p_E = p_T + p_w$$

STRESS IN SOIL WITHOUT SEEPAGE

Consider the soil layer shown



At Point A:

Total stress, $p_T = \gamma_m\ h_4 + q$

Neutral stress, $p_w = 0$

Effective stress, $p_E = p_T - p_w$

At point B:

Total stress, $p_T = \gamma_{sat1} h_5 + \gamma_m h_1 + q$

Neutral stress, $p_w = \gamma_w\ h_5$

Effective stress, $p_E = p_T - p_w$

or $p_E = \gamma_{b1}\ h_5 + \gamma_m\ h_1 + q$

At point C:

Total stress, $p_T = \gamma_{sat3}\ h_3 + \gamma_{sat1}\ h_2 + \gamma_m\ h_1 + q$

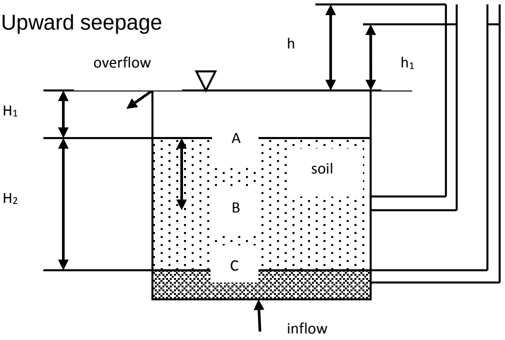
Neutral stress, $p_w = \gamma_w h_6$

Effective stress, $p_E = p_T - p_w$

$p_E = \gamma_{b2}\ h_3 + \gamma_{b1} h_2 + \gamma_m h_1 + q$

STRESS IN SATURATED SOIL WITH SEEPAGE

Upward seepage



Hydraulic gradient, $i = h/H_2$

$$h_1 = i \times z_1 = i(h/H_2)$$

At Point A:

$$p_T = \gamma_w H_1$$
$$p_E = \gamma_w H_1$$
$$p_E = p_T - p_w = 0$$

At Point B:

$$p_T = \gamma_{sat} z_1 + \gamma_w H_1$$
$$p_w = \gamma_w(z_1 + H_1 + h_1)$$
$$p_E = p_T - p_w = \gamma_b z_1 - \gamma_w h_1$$

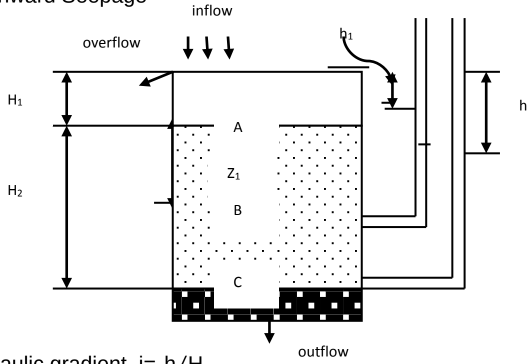
At Point C:

$$p_T = \gamma_{sat} H_2 + \gamma_w H_1$$
$$p_w = \gamma_w (H_2 + H_1 + h)$$
$$p_E = p_T - p_w = \gamma_b H_2 - \gamma_w h$$

The seepage force per unit volume of soil is:

$$F = i\,\gamma_w$$

Downward Seepage



Hydraulic gradient, $i = h/H_2$

$$h_1 = i \times z_1 = i\,(h/H_2)$$

At Point A:

$$p_T = \gamma_w H_1$$
$$p_E = \gamma_w H_1$$
$$p_E = p_T - p_w = 0$$

At Point B:

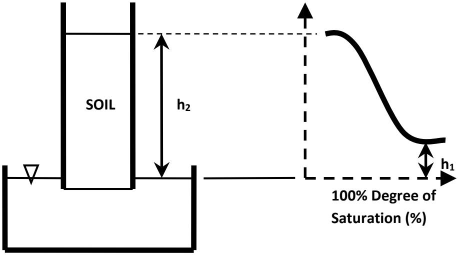
$$p_T = \gamma_{sat}\, z_1 + \gamma_w\, H_1$$
$$p_w = \gamma_w (z_1 + H_1 - h_1)$$
$$p_E = p_T - p_w = \gamma_b z_1 + \gamma_w h_1$$

At Point C:

$$p_T = \gamma_{sat} H_2 + \gamma_w H_1$$
$$p_w = \gamma_w (H_2 + H_1 - h)$$
$$p_E = p_T - p_w = \gamma_b H_2 + \gamma_w h$$

EFFECT OF CAPILLARY RISE TO SOIL STRESS

Capillary rise in soil is demonstrated on the following figure. A sandy soil is placed in contact with water. After a certain period, water rises and the variation of the degree of saturation with the height of the soil column caused by capillary rise is approximately given in the figure.



Variation of degree of saturation with height

The degree of saturation is about 100% up to a height h_1. Beyond the height h_1, water can occupy only the smaller voids, hence the degree of saturation is less than 100%

The approximate height of capillary rise is given by Hazen as:

$$\boxed{h_2 = \frac{C}{eD_{10}}}$$

Where D_{10} = effective grain size, e = void ratio, and C = a constant that varies from 10 to 50 mm^2

The pore water pressure, p_w, at a point in the layer of soil fully saturated by capillary rise is:

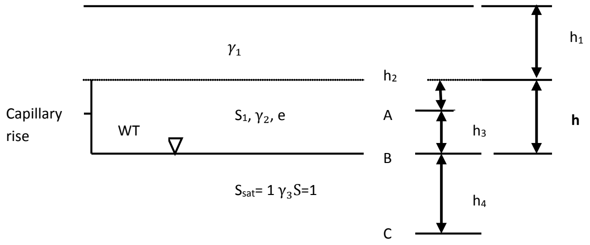
$$\boxed{p_w = -\gamma_w h}$$

Where h is the height of the point under consideration measured from the ground water table.

If a partial saturation is caused by capillary action, the pore water pressure, p_w, can be approximate as:

$$\boxed{p_w = -S\,\gamma_w h}$$

Where S is the degree of saturation at the point under consideration



For the soil shown above

At point A:

Total stress, $p_T = \gamma_1\ h_1 + \gamma_2\ h_2$

Pore water stress, $p_w = -S_1\ \gamma_w\ h_3$

At point B:

Total stress, $p_T = \gamma_1\ h_1 + \gamma_2\ h$

Pore water stress, $p_w = 0$

At point C:

Total stress, $p_T = \gamma_1\ h_1 + \gamma_2\ h + \gamma_3\ h_4$

Pore water stress, $p_w = \gamma_w\ h_4$

COMPRESSIBILITY OF SOIL

The increase in stressed caused by foundation and other loads compresses a soil layer. This compression is caused by (1) deformation of soil particles, (2) relocation of soil particles, and (3) expulsion of water or air from the void spaces.

Soil settlement may be divided into three categories:

1. Immediate settlement – caused by the elastic deformation of dry, moist, and saturated soils, without any change in moisture content.
2. Primary consolidation settlement – caused by a volume change in saturated cohesive soils due to expulsion of water that occupies the void spaces.
3. Secondary consolidation settlement – caused by plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

SETTLEMENT FROM ONE DIMENTIONAL PRIMARY CONSOLIDATION

Basic Settlement Formula

$$H = H_s(1+e); \qquad H_s = \frac{H}{1+e}$$

$$H' = H_s(1+e'); \qquad H' = \frac{H}{1+e}(1+e')$$

$$\Delta H = H - H'$$

$$\Delta H = H - \frac{H}{1+e}(1+e')$$

$$\Delta H = H\frac{1+e-(1+e')}{1+e} = \frac{e-e'}{1+e}$$

$$\boxed{\Delta H = \frac{H(e_o - e')}{1+e_o} = H\frac{\Delta e}{1+e_o}}$$

Where:
H = thickness of stratum
e_o = void ratio before the vertical load is applied
e' = void ratio after the vertical load is applied

PRIMARY CONSOLIDATION SETTLEMENT OF NORMALLY CONSOLIDATED FINE-GRAINED SOILS

$$\boxed{\Delta H = H \frac{C_c}{1+e^o} \log \frac{p_f}{p_o}}$$

Where:
H = thickness of stratum
C_c = compression index
e_o = initial void ratio
p_o = initial vertical effective soil stress
p_f = final vertical effective soil stress
$p_f = p_o + \Delta p$

PRIMARY CONSOLIDATION SETTLEMENT OF OVERCONSOLIDATED FINE-GRAINED SOILS

When $p_f < p_c$

$$\boxed{\Delta H = H \frac{C_s}{1+e^o} \log \frac{p_f}{p_o}}$$

When $p_f > p_c$

$$\boxed{\Delta H = H\frac{C_s}{1+e^o}\log\frac{p_c}{p_o} + H\frac{C_c}{1+e^o}\log\frac{p_f}{p_c}}$$

Where:
C_s= swell index
p_c = preconsolidation pressure

OVERCONSOLIDATION RATIO, OCR

$$\boxed{OCR = \frac{p_c}{p_o}}$$

Where:
p_c = preconsolidation stress (past maximum vertical effective stress)
p_o = overburden effective stress (concurrent vertical effective stress)

If OCR = 1, the soil normally consolidated soil

COMPRESSION INDEX, C_c:

Skempton:
For remolded clay:

$$\boxed{C_c = 0.007(LL - 7\%)}$$

For undisturbed clay:

$$\boxed{C_c = 0.009(LL - 10\%)}$$

Rendon-Herreo:

$$\boxed{C_c = 0.141\, G^{1.2} \left(\frac{1+ e_o}{G}\right)^{2.38}}$$

Nishida:
All clays

$$\boxed{C_c = 1.15\,(e_o - 0.27)}$$

SWELL INDEX, C_s:
The swell index is smaller in magnitude than the compression index. In most cases,

$$\boxed{C_s \cong \frac{1}{5}C_c \text{ to } \frac{1}{10}C_c}$$

Nagaraj and Murty:

$$\boxed{C_s = 0.0463\, \frac{LL\%}{100} \times G}$$

SETTLEMENT FROM SECONDARY CONSOLIDATION SECONDARY CONSOLIDATION CAN BE CALCULATED AS:

$$H_s = C'_{\alpha}\, H \, \log\left(\frac{t_2}{t_1}\right)$$

$$C'_{\alpha} = \frac{C_{\alpha}}{1+e_p}; \; C_{\alpha} = \frac{\Delta e}{\log t_2 - \log t_1}$$

Where

C_{α}=secondary compression index

Δe = change in void ratio

t_1=time for completion of primary settlement

t_2=time after completion of primary settlement,
where settlement is required

e_p = void ratio at the end of primary consolidation

$e_p = e_o - \Delta e$

H = thickness of clay layer

Calculation of Consolidation Settlement under a Foundation

The increase in vertical stress caused by a load applied over a limited area decreases by depth. To estimate the one-dimensional settlement of a foundation, we can use the equations of this section. However, the increase in of stress Δp should be the average increase in pressure below the center of the foundation.

Assuming the pressure increase varies parabolically, the average pressure may be estimated as:

$$\Delta p_{ave} = \frac{\Delta p_1 + 4\Delta p_m + \Delta p_b}{6}$$

Where:
Δp_1=increase in pressure at the top of the layer
Δp_m=increase in pressure at the middle of the layer
Δp_b=increase in pressure at the bottom of the layer

TIME RATE OF CONSOLIDATION

The time is required to achieve a certain degree of consolidation U is evaluated as a function of the shortest drainage path within the compressible zone H_{dr}, coefficient of consolidation C_v, and the dimensionless time factor T_v.

$$t = T_v \frac{(H_{dr})^2}{C_v}$$

Where:
H_{dr} = one-half the thickness of the drainage layer if drainage occurs at the top and bottom of the layer (two-way drainage)
H_{dr} = thickness of the drainage layer if drainage occurs at the top of the bottom only (one-way layer)

Variation of T_v with U

U%	T_v	U%	T_v	U%	T_v	U%	T_v
1		26	0.0531	51	0.204	76	0.493
2	0.00008	27	0.0572	52	0.212	77	0.511
3	0.00030	28	0.0615	53	0.221	78	0.529
4	0.00071	29	0.0660	54	0.230	79	0.547
5	0.00126	30	0.0707	55	0.239	80	0.567
6	0.00196	31	0.0754	56	0.248	81	0.588
7	0.00283	32	0.0803	57	0.257	82	0.610
8	0.00502	33	0.0855	58	0.267	83	0.633
9	0.00636	34	0.0907	59	0.276	84	0.658
10	0.00785	35	0.0962	60	0.286	85	0.684
11	0.0095	36	0.102	61	0.297	86	0.712
12	0.0113	37	0.107	62	0.307	87	0.742
13	0.0133	38	0.113	63	0.318	88	0.774
14	0.0154	39	0.119	64	0.329	89	0.809
15	0.0177	40	0.126	65	0.344	90	0.848
16	0.0201	41	0.132	66	0.352	91	0.891
17	0.0227	42	0.138	67	0.364	92	0.938
18	0.0254	43	0.145	68	0.377	93	0.993
19	0.0283	44	0.152	69	0.390	94	1.055
20	0.0314	45	0.159	70	0.403	95	1.129
21	0.0346	46	0.170	71	0.417	96	1.219
22	0.0380	47	0.173	72	0.431	97	1.336
23	0.0415	48	0.181	73	0.446	98	1.500
24	0.0452	49	0.188	74	0.461	99	1.781
25	0.0491	50	0.197	75	0.477	100	infinity

The approximate values of time factor T_v are

For U = 0 to 60%

$$T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

For U > 60%

$$T_v = 1.781 - 0.933 \log(100 - U\%)$$

The time factor T_v provides a useful expression to estimate the settlement in the field from the results of a laboratory consolidation.

$$\frac{t_{field}}{t_{lab}} = \frac{(H_{dr\ field})^2}{(H_{dr\ lab})^2}$$

$$\text{Also, } \frac{t_1}{t_2} = \frac{U_1^2}{U_2^2}$$

Where t_1 = time to reach a consolidation of $U_1\%$
T_2 = time to reach a consolidation of $U_2\%$

The degree of consolidation at a distance z at any time is:

$$U_z = 1 - \frac{p_{wz}}{p_{wo}}$$

The average degree of consolidation for the entire depth of layer at any time is:

$$U_z = \frac{\Delta H_t}{\Delta H_{max}}$$

Where

p_{wz}=excess pore pressure at time t

p_{wo}=initial excess pore water pressure

ΔH_t =settlement of the layer at time t

ΔH_{max}=ultimate settlement of the layer from primary consolidation

Coefficient of Consolidation

$$\text{Root time method, } C_v = \frac{0.848(H_{dr})^2}{t_{90}}$$

$$\text{Log time method, } C_v = \frac{0.197(H_{dr})^2}{t_{50}}$$

Where:

t_{90}=time for 90% consolidation ($\sqrt{t}$ curve)

t_{50}=time for 50% consolidation (logt curve)

Coefficient of Volume compressibility, m_v

$$m_v = \frac{a_v}{1+e_{ave}} = \frac{(e_o - e)/\Delta p}{1+e_{ave}}$$

$$e_{ave} = \frac{e + e_o}{2}$$

The hydraulic conductivity of the layer for the loading range is:

$$\boxed{k = C_v \, m_v \, \gamma_w}$$

Where e_o= initial void ratio
e = final void ratio
Δp = rise in pressure

IMMEDIATE SETTLEMENT

Immediate or elastic settlement of foundation occurs directly after application of a load, without change in moisture content. This depends on the flexibility of the foundation and the type of material on which it is resting.

Immediate settlement of foundations resting on the ground surface of an elastic material of finite thickness is given by:

$$\boxed{\Delta H_1 = pB \, \frac{1 - \mu^2}{E} \, I_f}$$

Where:
p = net pressure applied in kPa or psf
B = width or diameter of foundation in m or feet
μ = Poisson's Ratio
E = modulus of elasticity of soil in kPa or psf
I_f=influence factor (dimensionless)

The influence factor for the corner of a flexible rectangular footing given as:

$$I_f = \frac{1}{\pi}\left[m_1 \ln\left(\frac{1 + \sqrt{1 + m_1^2}}{m_1}\right) + \ln\left(m_1 + \sqrt{1 + m_1^2}\right)\right]$$

Influence Factors for Foundation

Shape	m_1	I_f		
		Flexible		Rigid
		Center	Corner	
Circle	-	1.00	0.64	0.79
Rectangle	1	1.12	0.56	0.88
	1.5	1.36	0.68	1.07
	2	1.53	0.77	1.21
	3	1.78	0.89	1.42
	5	2.10	1.05	1.70
	10	2.54	1.27	2.10
	20	2.99	1.49	2.46
	50	3.57	1.8	3.00
	100	4.01	2.0	3.43

Where m_1 = length of foundation / width of foundation

Values of Modulus of Elasticity

Type of soil	E	
	psi	kPa
Soft clay	250-500	1,725-3,450
Hard clay	850-2,000	5,865-13,800
Loose clay	1,500-4,000	10,350-27,600
Dense clay	5,000-10,000	34,500-69,000

Values of Poisson's Ratio

Type of Soil	Poisson's Ratio
Loose sand	0.2-0.4
Medium sand	0.25-0.4
Dense sand	0.3-0.45
Silty sand	0.2-0.4
Soft clay	0.25-0.25
Medium clay	0.2-0.5

TOTAL SETTLEMENT OF FOUNDATION

The total settlement of a foundation is the sum of the primary, secondary, and immediate settlement.

$$\boxed{\Delta H_T = \Delta H + \Delta H_s + \Delta H_i}$$

SHEAR STRENGTH OF SOIL

The shear strength of soil may be attributed to three basic components.

1. Frictional resistance to sliding between solid particles
2. Cohesion and adhesion between particles
3. Interlocking and bridging of solid particles to resist deformation

MOHR-COULOMB FAILURE CRITERIA

A material fails because of a critical combination of normal stress and shearing stress, and not from either maximum normal shear stress alone. This theory was presented by Mohr. Thus, a failure can be expressed as a function of normal and shearing stress as follows.

$$\boxed{\tau_f = f(\sigma)}$$

For most soil mechanics problems, Coulomb suggested that the shear stress on the failure plane can be expressed as a linear function of normal stress. This relationship is known as Mohr-Coulomb failure criteria and can written as:

$$\boxed{\tau_f = C + \sigma \tan\emptyset}$$

Where C = cohesion

ϕ = angle of internal friction

These functions are shown in the figure below. The significance of the failure envelope is as follows. If the normal and shearing stress on a plane in a soil mass are such that they plot as point X, shear failure will not occur along that plane. If it plots at Y, shear failure will occur along that plane because it plots along that plane. Point Z cannot exist because it plots above the failure envelope and shear failure would have occurred already

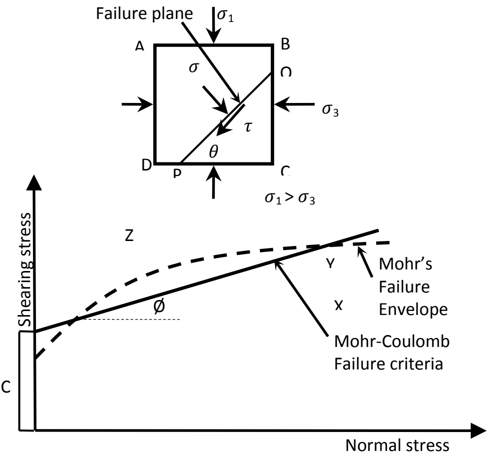
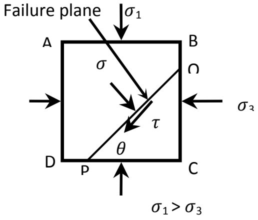


Figure: Mohr's failure envelope and Mohr-coulomb failure criteria



Applied stresses on soil

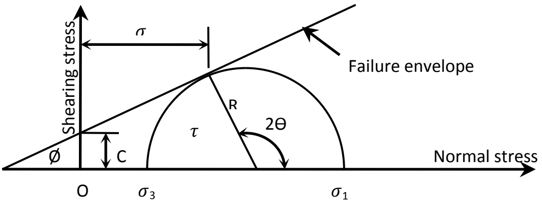


Figure: Mohr's circle

$$\theta = 45^{\circ} + \frac{\varnothing}{2}$$

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\varnothing}{2}\right) + 2c\tan\left(45 + \frac{\varnothing}{2}\right)$$

TRIAXIAL SHEAR TEST (SINGLE TEST)

Cohesionless Soil

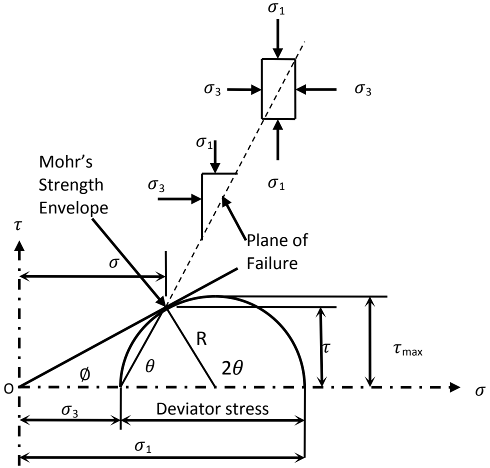


Figure: Single test on cohesionless soil

The following equation can be derived from the figure:

$$\tau = \sigma \tan \phi = R \sin 2\theta$$
$$R = {}^{1}/_{2}\,(\sigma_1 - \sigma_3) = \tau_{max}$$
$$\sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$
$$\theta = 45^{\circ} + \phi/2$$

COHESIVE SOIL

$$\tau = C + \sigma \tan\phi = R \sin 2\theta$$

Where:
σ_1 =Major principal stress at failure
σ_3=Minor principal stress at failure
τ **=** Shear stress
C = Cohesion of soil
ϕ=angle of internal friction
θ **=** angle that the failure plane makes with the major principal plane.

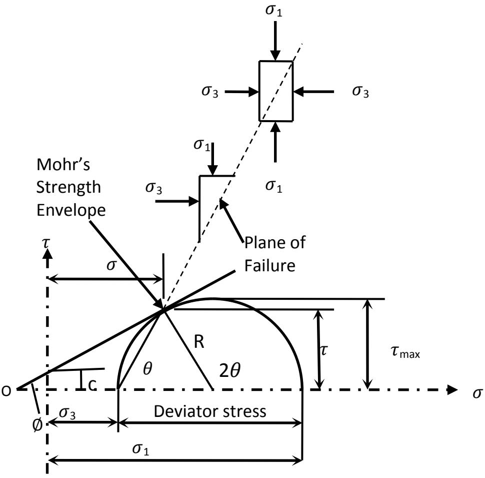
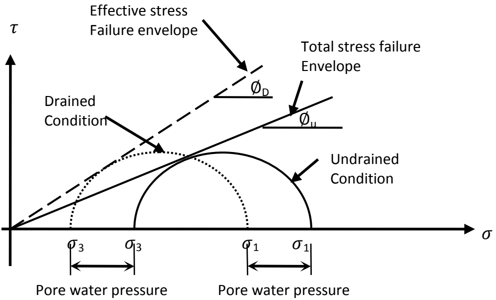


Figure: Single test on cohesive soil

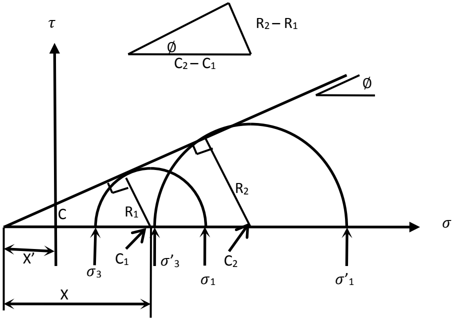
Drained and Undrained Triaxial Test

For drained triaxial test σ'_1 and σ'_3 are taken as the effective principal stresses. For undrained triaxial test, σ_1 and σ_3 are taken as the total principal stresses.



Drained and undrained condition

TRIAXIAL TEST (SERIES)



$$R_1 = \frac{\sigma_1 - \sigma_3}{2};\ R_2 = \frac{\sigma'_1 - \sigma'_3}{2}$$

$$C_1 = \frac{\sigma_1 + \sigma_3}{2};\ C_2 = \frac{\sigma'_1 + \sigma'_3}{2}$$

$$\sin\emptyset = \frac{R_2 - R_1}{C_2 - C_1};\ c = x' \tan\emptyset$$

$$x' = x - C_1 \text{ and } x = \frac{R_1}{\sin\emptyset}$$

$$c = R_1\cos\emptyset - (C_1 - R_1\sin\emptyset)\tan\emptyset$$

$$c = R_2\cos\emptyset - (C_2 - R_2\sin\emptyset)\tan\emptyset$$

Unconfined Compression Test (Uniaxial)

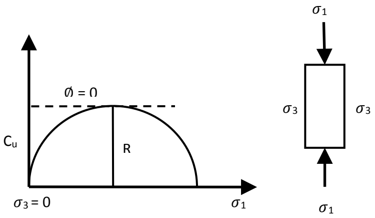


Figure: Unconfined compression test

$$C_u = R = \frac{\sigma_1}{2}$$
$$q_u = 2C_u$$

Where q_u = unconfined compression strength

DIRECT SHEAR TEST

Direct shear test is the simplest form of shear test. The test equipment consists of a metal shear box in which the soil sample is placed. The sizes of the sample used are usually 50mm x 50mm or 100mm x 100mm across and about 25mm high. The box is split horizontally into halves. A normal force is applied from the top of the shear box. Shear force is applied by moving half of the box relative to the other to cause failure in the soil sample.

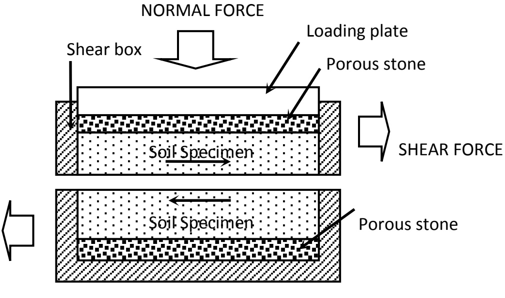


Figure: Direct shear test arrangement

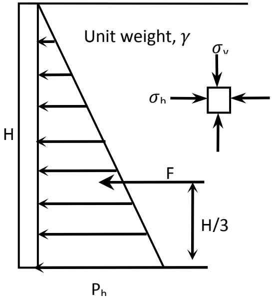
$$\text{Normal stress, } \sigma = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

$$\text{Shear stress, } \tau = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$

LATERAL EARTH PRESSURE

Active earth pressure coefficient, K_a – the ratio between the lateral and vertical principal effective stresses when an earth retaining structure moves away (by a small amount) from a retained soil.

Passive earth pressure coefficient, K_p – the ratio between the lateral and vertical principal effective stresses when an earth retaining structure is forced against a soil mass.



Earth Pressure at Rest

If the retaining structure does not move either to the right or to the left of its initial position, the soil mass will be in a state of elastic equilibrium, meaning, the horizontal strain is zero. The ratio of the horizontal stress to the vertical stress is called the coefficient of earth pressure at rest, K_o.

$$\boxed{K_o = \frac{\sigma_h}{\sigma_v} = 1 - \sin\emptyset}$$

Where Ø is the drained friction angle

For dense sand backfill:

$$\boxed{K_o = (1 - \sin\emptyset) + \left(\frac{\gamma_d}{\gamma_{dmin}}\right)5.5}$$

Where:
γ_d =actual compacted dry unit weight of the sand behind the wall
γ_{dmin} = dry unit weight of t5he sand in the loosest state

For fine-grained normally consolidated soils:

$$\boxed{K_o = 0.4\,4 + 0.42\,(PI\%/100)}$$

For overconsolidated clays:

$$K_{o\,(\text{overconsolidated})} = K_{o\,(\text{normally consolidated})}$$

$$OCR = \frac{\text{Preconsolidation pressure}}{\text{Present effective overburden pressure}}$$

$$p_h = K_o\,\gamma\,H$$
$$F = {}^{1}/_{2}\ K_o \gamma H^2$$

RANKINES THEORY

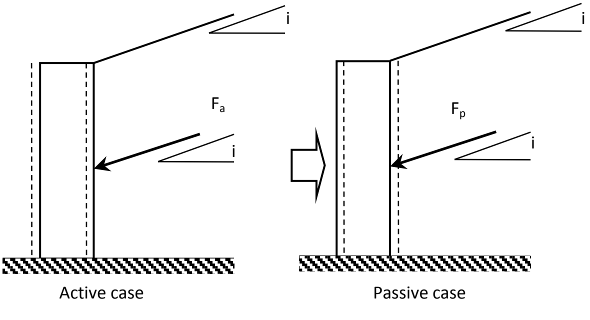


Figure: Vertical face and inclined backfill

Coefficient of active pressure:

$$K_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \emptyset}}{\cos i + \sqrt{\cos^2 i - \cos^2 \emptyset}} \cos i$$

Coefficient of passive pressure:

$$K_p = \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \emptyset}}{\cos i - \sqrt{\cos^2 i - \cos^2 \emptyset}} \cos i$$

Rankine's Theory (for horizontal backfill)

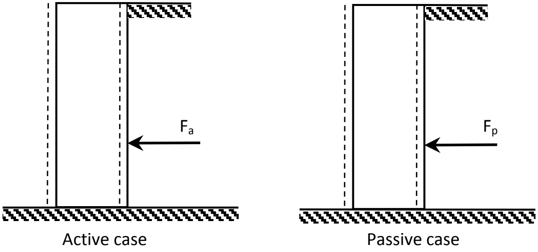


Figure: Vertical face and horizontal backfill

Coefficient of active pressure:

$$\boxed{K_a = \frac{1 - \sin\phi}{1 + \sin\phi}}$$

Coefficient of passive pressure

$$\boxed{K_a = \frac{1 + \sin\phi}{1 - \sin\phi}}$$

COULOMB'S THEORY

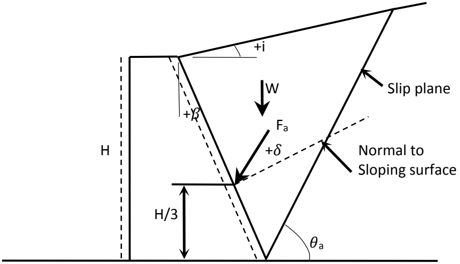


Figure: Wall sloping face (active case)

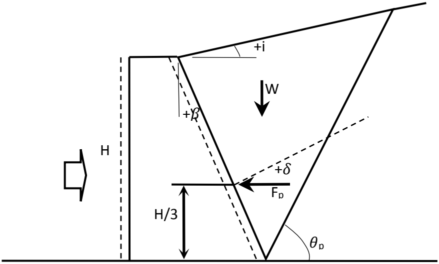


Figure: Wall sloping face (passive case)

Because of frictional resistance to sliding at the face of the wall, F_a and F_p is inclined at an angle of δ with the normal to the wall, where δ is the angle of wall friction

ACTIVE PRESSURE COEFFICIENT

$$K_a = \frac{\cos^2(\phi - \beta)}{\cos^2\beta\cos(\beta + \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - i)}{\cos(\beta + \delta)\cos(\beta - i)}}\right]^2}$$

The inclination of the slip plane to the horizontal is:

$$\tan\theta_a = \frac{\sqrt{\sin\phi \cos\delta}}{\cos\phi\sqrt{\sin(\phi + \delta)}} + \tan\phi$$

The effect of wall friction on K_a is small, and is usually neglected. For $\delta = 0$:

$$K_a = \frac{\cos^2(\phi - \beta)}{\cos^3\beta\left[1 + \sqrt{\frac{\sin\phi\sin(\phi - i)}{\cos\beta\cos(\beta - i)}}\right]^2}$$

When i=0 and δ=0:

$$K_a = \frac{\cos^2(\phi - \beta)}{\cos^3\beta\left[1 + \sqrt{\frac{\sin\phi\sin(\phi - i)}{\cos\beta\cos(\beta - i)}}\right]^2}$$

For wall with vertical back face supporting granular soil backfill with horizontal surface (i.e. i = 0° and β = 0°), the above equation yields

$$K_a = \frac{\cos^2(\phi - \beta)}{\cos^3\beta\left[1 + \sqrt{\dfrac{\sin\phi\sin(\phi - i)}{\cos\beta\cos(\beta - i)}}\right]^2}$$

PASSIVE PRESSURE COEFFICIENT

$$K_p = \frac{\cos^2(\phi + \beta)}{\cos^2\beta\cos(\beta - \delta)\left[1 - \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi + i)}{\cos(\beta - \delta)\cos(\beta - i)}}\right]^2}$$

The inclination of the slip plane to the horizontal is

$$\tan\theta_p = \frac{\sqrt{\sin\phi\cos\delta}}{\cos\phi\sqrt{\sin(\phi + \delta)}} - \tan\phi$$

For frictionless wall with vertical back face supporting granular soil backfill with horizontal surface (i.e.$\delta = 0^o$, $i = 0^o$ and $\beta = 0^o$):

$$K_p = \frac{1 + \sin\theta}{1 - \sin\theta}$$

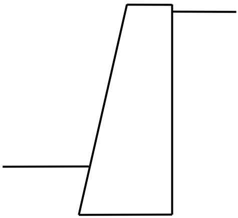
The critical value of θ is:

$$\theta = \theta_{cr} = 45^\circ + \phi/2$$

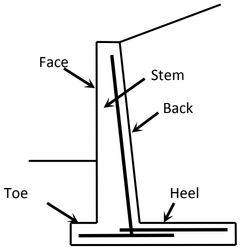
RETAINING WALLS

A retaining wall may be defined as a structure whose primary purpose is to prevent lateral movement of earth or some other material. For some special causes, as in basement walls or bridge abutments, a retaining wall may also have a function of supporting vertical loads.

Types of Retaining Walls



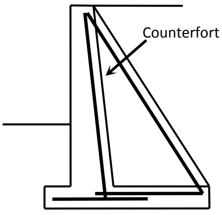
(a) Gravity retaining wall



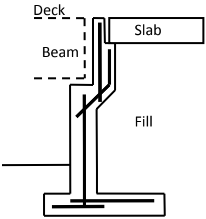
(b) T-shaped retaining wall

Gravity retaining wall is usually built of plain concrete. This type of wall depends only on its own weight for stability, and hence, its height is subject to some definite practical limits.

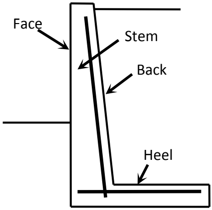
Semi-gravity wall is in essence a gravity wall that has been given a wider base (a toe or heel or both) to increase its stability. Some reinforcement is usually necessary for this type of wall.



(d) Counterfort retaining wall



(b) Counterfort retaining wall



(c) L-shaped retaining wall

T-shaped wall is perhaps the most common cantilever wall. For this type of wall, the weight of the earth in the back of the stem (the backfill) contributes to stability.

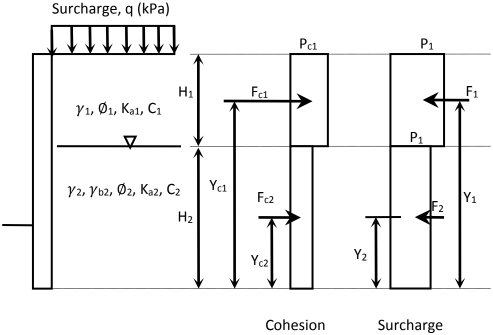
L-shaped wall is frequently used when property line restrictions forbid the use of a T-shaped wall. On the other hand, when it is not feasible (due to construction limitation) to excavate for a heel, a reversed L-shaped may served the need.

Counterfort retaining wall, consists of three main component: base, stem, and intermittent vertical ribs called counterforts, which tie the base and the stem together. These ribs, which acts as tension ties, transform the stem and heel into continuous slabs supported on three sides – at two adjacent counterforts and at the base of the stem.

Buttressed wall is constructed by placing the ribs on the front face of the stem where they act in compression.

Bridge abutment is a retaining wall, generally short and typically accompanied by wing walls.

ACTIVE PRESSURE ON WALL



$$K_a = \frac{1-\sin\emptyset}{1+\sin\emptyset} \quad \text{(Rankine or Coulomb)}$$

Cohesion:

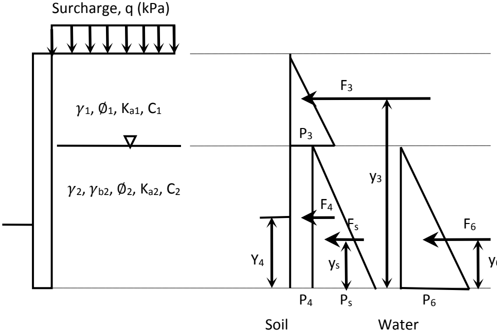
$p_{c1} = 2c_1\sqrt{K_{a1}}$; $F_{c1} = p_{c1} \times H_1$

$p_{c2} = 2c_2\sqrt{K_{a2}}$; $F_{c2} = p_{c2} \times H_2$

Surcharge:

$p_1 = K_{a1}\, q$; $F_1 = p_1 \times H_1$

$p_2 = K_{a2}\, q$; $F_2 = p_2 \times H_2$



Soil:

$p_3 = K_{a1}\,\gamma_1\,H_1$; $F_3 = {}^1/_2\,p_3H_1$

$p_4 = K_{a2}\,\gamma_1\,H_1$; $F_4 = p_4H_2$

$p_5 = K_{a2}\,\gamma_{b2}\,H_2$; $F_5 = {}^1/_2\,p_5H_2$

Water:

$p_6 = \gamma_w\,H_2$; $F_5 = {}^1/_2\,p_5H_2$

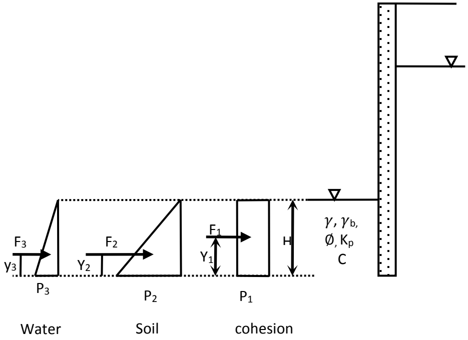
Total active force:

$F_a = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 - F_{c1} - F_{c2}$

Total active moment:

$M_a = F_1y_1 + F_2y_2 + F_3y_3 + F_4y_4 + F_5y_5 + F_6y_6 - F_{c1}y_{c1} - F_{c2}y_{c2}$

PASSIVE PRESSURE ON WALL



$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \text{ (Rankine or Coulomb)}$$

Cohesion:

$$p_1 = 2c\sqrt{K_p} \quad ; \quad F_1 = p_1 H$$

Soil:

$$p_2 = K_p \gamma_b H \quad ; \quad F_2 = \frac{1}{2} p_2 H$$

Water:

$$p_3 = \gamma_w H \quad ; \quad F_3 = \frac{1}{2} p_3 H$$

Total passive resistance, $F_p = F_1 + F_2 + F_3$
Total passive moment, $M_p = F_1y_1 + F_2y_2 + F_3y_3$

FACTORS OF SAFETY

The structural elements of the wall should be so proportioned that the following safety factors are realized:

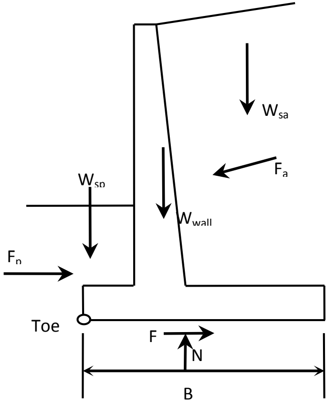


Figure: Forces acting on wall

Factor of Safety Against Sliding:

$$FS_S = \frac{\text{Resisting forces}}{\text{Active forces}}$$

For granular backfill, $FS_S \geq 1.5$
For cohesive backfill, $FS_S \geq 2.0$

Factor of Safety Against Overturning About the Toe:

$$\boxed{FS_S = \frac{\text{Stabilizing moment}}{\text{Overturning moment}}}$$

For granular backfill, $FS_S \geq 1.5$

For cohesive backfill, $FS_S \geq 2.0$

The horizontal components of the lateral forces tends to force the wall to slide along its base. The resisting force is provided by the horizontal forces composed of friction and adhesion, and by passive resistance of soil in front of the wall. The passive resistance is not to be counted on if there is a chance that the soil kin front of the wall may be eroded or excavated during the life of the wall.

The force F at the base of the wall consist of the friction and cohesion. It is given by:

$$\boxed{F = \mu N + C_b B}$$

Where N is the normal reaction, μ is the coefficient of friction c_b is the base cohesion, and B is the base width of wall.

Commonly assumed values of μ and c_b are as follows:

$$\tan\phi > \mu > (2/3)\tan\phi$$

$$0.5c \leq c_b \leq 0.75c$$

<u>Pressure Distribution at Base of Wall</u>

The actual bearing pressure on the base of the wall is a combination of normal forces and the effects of moments.

$$R_y = \sum F_y$$

$$R_y x = RM - OM$$

$$e = \frac{B}{2} - x$$

Where:
RM = righting or stabilizing moments
OM = overturning moments

Note that in computing RM and R_y, the passive resistance is not to be counted on if there is a chance that the soil in front of the wall may be eroded or excavated during the life of the wall.

When $e \leq B/6$

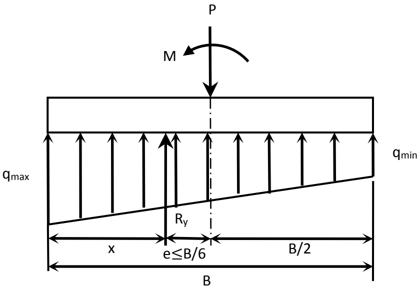


Figure: stress distribution at base of wall when e≤ B/6

Considering 1m length of wall

$$q_{min} = -\frac{R_y}{B}\left(1 \pm \frac{6e}{B}\right)$$

When e > B/6

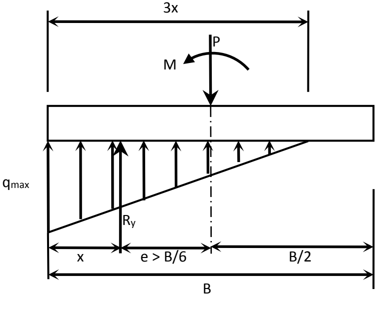


Figure: stress distribution at base of wall when e > B/6

Considering 1m length of wall

$$\boxed{q_{max} = -\frac{2R_1}{3x}}$$

Lateral Pressure on Retaining Walls Due to Point-load Surcharge

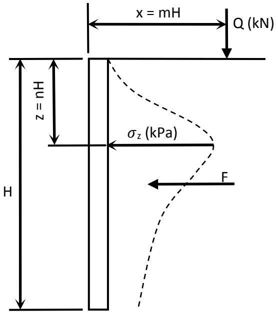


Figure: Stress on wall caused by a point load

The lateral stress on the wall induced by a point-load surcharge is given by:

$$\text{For } m > 0.4;\ \sigma_x = \frac{1.77Q}{H^2}\frac{m^2n^2}{(m^2 + n^2)^3}$$

$$\text{For } m \le 0.4;\ \sigma_x = \frac{0.28Q}{H^2}\frac{n^2}{(0.16 + n^2)^3}$$

Where Q is the point load (kN or lbs), H is the height of wall (m or feet), and σ_x is the stress (kPa or psf)
The force F per unit length of wall caused by the point load can be obtained by approximating the area of the shaded portion using trapezoidal rule or Simpson's one-third rule.
Lateral Pressure on Retaining Walls Due to Line-load Surcharge

The lateral stress on the wall induced by a line-load surcharge is given by:

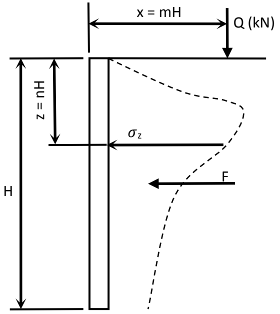


Figure: Stress on wall caused by a line load

$$\text{For } m > 0.4;\ \sigma_x = \frac{4q}{\pi H}\frac{m^2 n}{(m^2+n^2)^3}$$

$$\text{For } m \leq 0.4;\ \sigma_x = \frac{0.203q}{H}\frac{n}{(0.16+n^2)^2}$$

Where q is the line load (kN/m or lbs/ft), H is the height of wall (m or feet), and σ_x is the stress (kPa or psf)

The force F per unit length of wall caused by the strip load can be obtained by approximating the area of the shaded portion using trapezoidal rule or Simpson's one-third rule.

Lateral Pressure on Retaining Walls Due to Strip-Load Surcharge

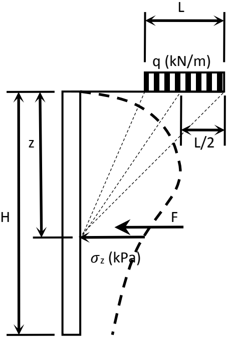


Figure: Stress on wall caused by a strip load

The lateral stress on the wall induced by a strip-load surcharge is given by:

$$\sigma_x = \frac{2q}{H}(\beta - \sin\beta \cos 2\alpha)$$

The force F per unit length of wall caused by the strip load can be obtained by approximating the area of the shaded portion using trapezoidal rule, or Simpson's one-third rule, or by integration of σ_x with limits from 0 to H.

BRACED CUTS

Bracing is used when temporary trenches for water, sanitary, and other lines are opened in soil. A braced cut is an excavation in which the active earth pressure from one bulkhead. The box-shoring and close-sheeting methods of support are shown in the figure below

The load is transferred to the struts at various points, so the triangular active pressure distribution does not develop. Since struts are installed as the excavation goes down, the upper part of the wall deflects very little due to the strut restraint. The pressure on the upper part of the wall is considerably higher than is predicted by the active earth pressure equations.

The soil removed from the excavation is known as the spoils. Spoils should be placed far enough from the edge of the cut so that they do not produce a surcharge lateral loading

The bottom of the excavation is referred to as the base of the cut, mudline, dredge line, and toe of the excavation. Excavations below the water table should be dewatered prior to cutting.

BRACED CUTS IN SAND

The analysis of the braced cuts is approximate due to the extensive bending of the sheeting. For drained san, the pressure distribution is approximately uniform with depth.

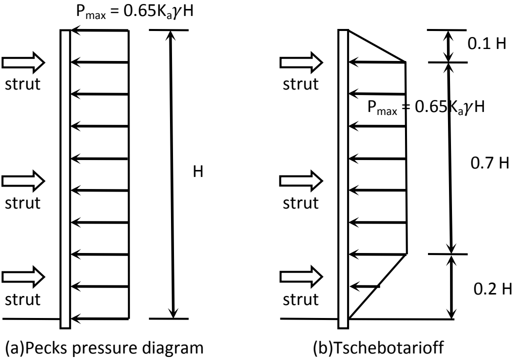


Figure: Pressure diagram for design of bracing system

BRACED CUTS IN STIFF CLAY

For undrained clay $\emptyset = 0^\circ$. In this case, the lateral pressure distribution depends on the average undrained shear strength (cohesion) of the clay. If $\gamma H/c \leq 4$, the clay is stiff and the pressure distribution is given as:

$$\boxed{p_{max} = 0.2\,\gamma\,H \text{ to } 0.4\,\gamma\,H}$$

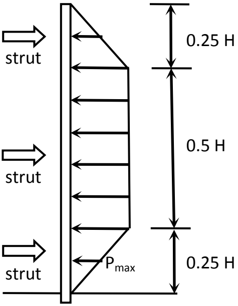


Figure: Peck's pressure diagram for stiff clay

Except when the cut is underlain by deep, soft, normally consolidated clay, the maximum pressure can be approximated as

$$\boxed{p_{max} = \left(1 - \frac{4c}{\gamma H}\right)\gamma H}$$

BRACED CUTS IN SOFT CLAY

If $\gamma H/c \geq 6$, the clay is soft and the lateral pressure distribution will be shown below

$$\boxed{p_{max} = \left(1 - \frac{4c}{\gamma H}\right)\gamma H}$$

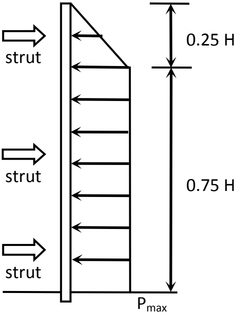


Figure: Peck’s pressure diagram for soft clay

For cuts underlain by deep, soft, normally consolidated clays, the maximum pressure is:

$$\boxed{p_{max} = \gamma - 4c}$$

If $4 < \gamma H/c < 6$, the soft and stiff clay cases should both be evaluated, the case that results in greater pressure should be used when designing the bracing.

ANALYSIS OF STRUT REACTION

Since braced excavations with more than one strut are statically indeterminate, strut forces and sheet piling moments may be evaluated by assuming hinged beam action.

The strut load may be determined by assuming that the vertical members are hinged at each strut level except the topmost and the bottommost ones.

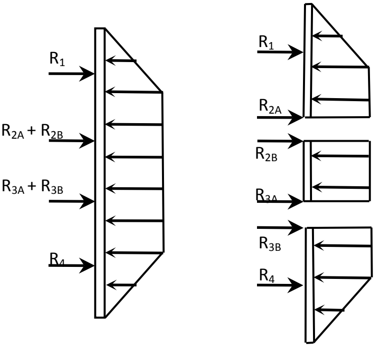


Figure: Determination of strut load

BEARING CAPACITY OF SOILS

DEFINITIONS

Foundation is that part of structure which transmits the building load directly into the underlying soil. If the soil conditions at the site are sufficiently strong and capable of supporting the required load, then shallow spread footings or mats can be used to transmit the load.

Footing is a foundation consisting of a small slab for transmitting the structure load to the underlying soil. Footings can be individual slabs supporting single columns or combined to support two or more columns, or be a long strip of concrete slab (width B to length L ratio is small, i.e., it approaches zero) supporting a load bearing wall, or a mat.

<u>Shallow foundation</u> is one in which the ratio of the embedment depth to the minimum plan dimension, which is usually the width, is $D_f/B \leq 2.5$.

<u>Embedment depth</u> (D_1) is the depth below the ground surface where the base of the foundation rests.

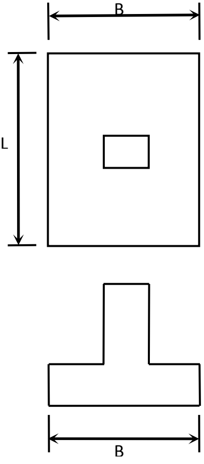
<u>Ultimate bearing capacity</u> (q_u) is the maximum pressure that the soil can support.

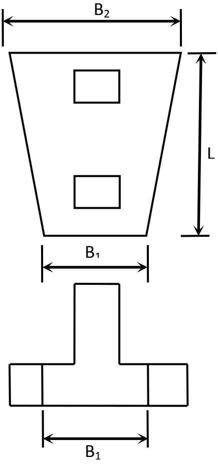
<u>Ultimate bearing capacity</u> (q_{ult}) is the maximum pressure that the soil can support above its current overburden pressure.

<u>Allowable bearing capacity or safe bearing capacity</u> (q_a) is the working pressure that would ensure a margin of safety against collapse of the structure from shear failure. The allowable bearing capacity is usually a fraction of the ultimate net bearing capacity.
<u>Overburden Pressure</u> q, is the pressure (effective stress) of the soil removed to place the footing.

<u>Factor of safety or safety factor</u> (FS) is the ratio of the ultimate net bearing capacity to the allowable bearing capacity or the applied maximum vertical stress. In geotechnical engineering, a factor of safety between 1.5 and 5 is used to calculate the allowable bearing capacity

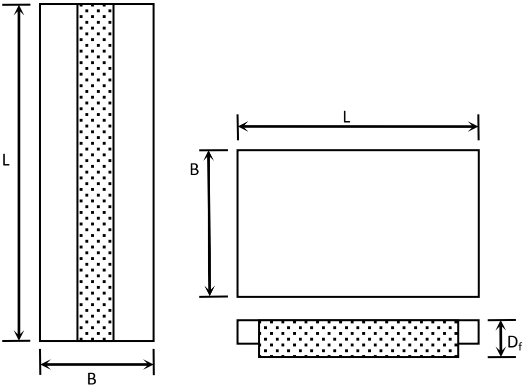
VARIOUS TYPES OF FOOTING ON SOIL





Combined footing

Isolated footing



Wall footing

Mat or Raft

BEARING CAPACITY ANALYSIS

Bearing capacity analysis is the method used to determine the ability of the soil to support the required load in a safe manner without gross distortion resulting from objectionable settlement. The ultimate bearing capacity (q_u) is defined as that pressure causing a shear failure of the supporting soil lying immediately below and adjacent to the footing. Generally three modes of failure have been identified:

1. General Shear Failure: a continuous failure surface develops between the edge of the footing and the ground surface. This type of failure is characterized by heaving at the ground surface accompanied by tilting

of the footing. It occurs in soil of low compressibility such as dense sand or stiff clay.

2. Local Shear Failure: a condition where significant compression of the soil occurs but only slight heave occurs at the ground surface. Tilting of the foundation is not expected. This type of failure occurs in highly compressible soil and ultimate bearing capacity is not well defined.
3. Punching Shear failure: a condition that occurs where there is relatively high compression of the soil underlying the footing with neither heaving at the ground surface nor tilting of the foundation. Large settlement is expected without a clearly defined ultimate bearing capacity. Punching will occur in low compressible soil if t5he foundation is located at a considerable depth below ground surface.

ULTIMATE SOIL BEARING CAPACITY

In general, the ultimate bearing capacity of soil is expressed as by:

$$\boxed{q_u = K_c\, c\, N_c + K_q\, q\, N_q + K_\gamma\, \gamma_e\, B\, N_\gamma}$$

Where:

q_u=ultimate bearing capacity

γ_e =unit weigth of the soil in kPa or pcf

B = width of footing in meter or feet

N_γ=factor for unit weight of soil

N_c=factor of soil cohesion

N_q=factor of overburden pressure
q = overburden pressure (effective stress)
K_c , K_q , K_γ =constant

TERZAGHI'S BEARING CAPACITY EQUATIONS

Terzaghi's bearing capacity equations are based on the following assumptions:

- Depth of foundation is less than or equal to its width
- No sliding occurs between foundation and soil(rough foundation)
- Soil beneath foundation is homogeneous semi-infinite mass
- Mohr-Coulomb model for soil
- General shear failure mode is the governing mode (but not the only mode)
- No soil consolidation occurs
- Foundation is very rigid relative to the soil
- Soil above bottom of foundation has no shear strength; is only a surcharge load against the overturning load
- Applied load is compressive and applied vertically to the centroid of the foundation
- No applied moments present

GENERAL SHEAR FAILURE:

Long Footings

$$\boxed{q_u = cN_c + qN_q + \tfrac{1}{2}\gamma_e BN_\gamma}$$

Square Footings

$$q_u = 1.3cN_c + qN_q + 0.4\gamma_e BN_\gamma$$

Circular Footings

$$q_u = 1.3cN_c + qN_q + 0.3\gamma_e BN_\gamma$$

Where:
γ_e =unit weight of soil at base of footing in kPa or pcf
B = width of footing in meter or feet
c = cohesion of soil in kPa of psf
N_γ=factor for unit weight
N_c =factor of soil cohesion
N_q =factor of overburden pressure
q = overburden pressure (effective stress) at base of footing
D_f =depth of footing in meter or feet

<u>LOCAL SHEAR FAILURE</u>

For local shear failure, it may be assumed that

$$C = \frac{2}{3}c$$
$$\tan \emptyset = \frac{2}{3}c$$

Long Footing (Strip Footing)

$$q_u = cN_c + qN_q + {}^1\!/_2\,\gamma_e B N_\gamma$$

Square Footing

$$q_u = 1.3cN_c + qN_q + 0.4\gamma_e B N_\gamma$$

Circular Footing

$$q_u = 1.3cN_c + qN_q + 0.3\gamma_e B N_\gamma$$

ALLOWABLE BEARING CAPACITY ANF FACTOR OF SAFETY

The allowable bearing capacity, q_a, is calculated by dividing the ultimate bearing capacity, q_u, by a factor of safety, FS. The factor of safety is intended to compensate for the assumptions made in developing the bearing capacity equations, soil variability, inaccurate soil data, and uncertainties of loads.

Gross Allowable Bearing Capacity:

$$q_{all} = \frac{q_u}{FS}$$

Net Allowable Bearing Capacity:

$$q_{all(net)} = \frac{q_{unet}}{FS}$$
$$q_{unet} = q_u - q$$

GROSS ALLOWABLE BEARING CAPACITY WITH FS WITH RESPECT TO SHEAR

The gross allowable bearing capacity using a factor of safety on shear strength of soil may be computed using the developed cohesion c_d and values of N_c, N_q, and N_γ derived using the developed angle of friction ϕ_d.

$$\text{Developed cohesion, } c_d = c/FS$$
$$\text{Developed angle of friction, } \emptyset_d = \tan^{-1}\left(\frac{\tan\emptyset}{FS}\right)$$

For example, on strip footing, $q_a = c_d N_c + qN_q + \frac{1}{2}\gamma_e B n_\gamma$

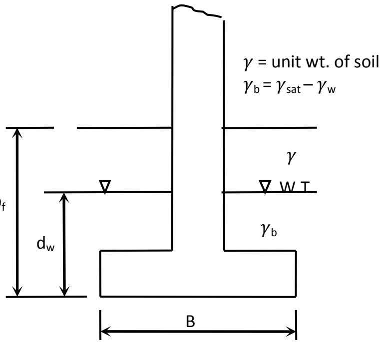
Alternatively, if the maximum applied foundation stress, $(f_a)_{max}$ is known, the factor of safety can be computed by replacing q_a by $(f_a)_{max}$.

$$FS = \frac{q_u}{(f_a)_{max} - q}; q < (f_a)_{max}$$

EFFECT OF WATER TABLE ON BEARING CAPACITY

The unit weight of soil used in the equations for bearing capacity are effective unit weights. With the rising water table, the subsoil becomes saturated and the unit weight of submerged soil is greatly reduced. The reduction of this unit weight results in a decrease in the ultimate bearing capacity of the soil.

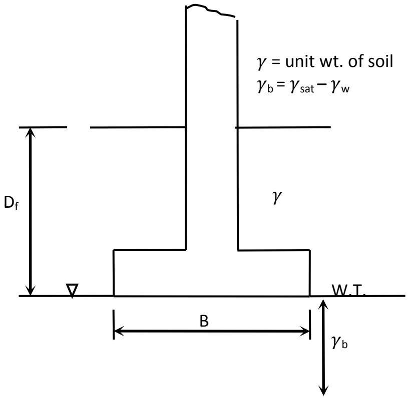
Groundwater level above base footing



$$q = \gamma(D_f - D_w) + \gamma_b d_w$$

$$\text{Unit Weight, } \gamma_e = \gamma_b$$

Groundwater level at the base of footing



overburden pressure, q= γD_t

Unit Weight, $\gamma_e = \gamma_b$

Groundwater level below the base of footing

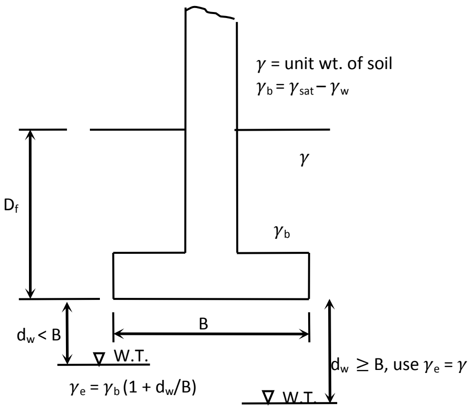
overburden pressure, q= γD_t

When $d_w < B$

$$\boxed{\gamma_e = \gamma_b\,(1 + d_w/B) = \text{approx.}}$$

When $d_w \geq B$

$$\gamma_e = \gamma$$



<u>MEYERHOF'S EQUATION</u>

(General Bearing capacity equation)

Vertical load:

$$q_u = cN_cS_cd_c + qN_qS_qd_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma$$

Inclined load:

$$q_u = cN_cS_cd_ci_c + qN_qS_qd_qi_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma i_\gamma$$

Bearing Capacity Factors:

$$N_d = e^{\pi \tan\emptyset}\tan^2\left(45^\circ + \emptyset/2\right)$$

$$N_c = \left(N_q - 1\right)\cot\emptyset$$

$$N_\gamma = \left(N_q - 1\right)\tan(1.4\emptyset)$$

BEARING CAPACITY FROM STANDARD PENETRATION TEST (SPT)

Allowable bearing capacity

$$q_a = 0.41N_{cor}\rho_{a\ (kpa)}$$
$$N_{cor} = C_NC_wN$$

Where:
N = standard penetration number
C_N=correction factor for over burden pressure

C_w = correction factor if the groundwater level is below the base of footing

PILES AND DEEP FOUNDATION

PILE CAPACITY FROM DRIVING DATA

(Dynamic Pile Formulas)

AASHTO Formula

$$\boxed{Q_u = \frac{2h(W_r + A_r p)}{s + 0.1}}$$

Recommended factor of safety = 6

Navy-McKay Formula

$$\boxed{Q_u = \frac{e_h E_h}{s\left(1 + 0.3\frac{W_p}{W_r}\right)} \quad ,\text{lbs}}$$

Recommended factor of safety = 6

Eytelwein Formula

$$\boxed{Q_u = \frac{e_h E_h}{s + 0.1\left(W_p / W_r\right)} \quad ,\ \text{lbs}}$$

Recommended factor of safety = 6

Where:
e_h = efficiency of hammer
A_r =ram cross-section, in^2
P = pressure, psi
E_h = rated hammer energy, in-lb
W_p = total weight of pile, pounds
W_r = weight of ram, pounds

THEORETICAL PILE CAPACITY

The ultimate load capacity Q_u consists of two parts. One part due to friction,called skin friction or shaft friction or side shear Q_f, and the other is due to end bearing at the base or tip of the pile Q_b.

$$\boxed{Q_u = Q_f + Q_b}$$

Where:
Q_f = skin/shaft friction or side shear (ultimate)
Q_b = end bearing or point resistance (ultimate)

The Alpha Method

The alpha method determines the adhesion factor, α as the ratio of the skin friction factor, f_s , to the undrained shear strength (cohesion), c_u.

$$\boxed{\begin{array}{c} Q_f = \alpha\, c_u PL \\ Q_b = f_b\, A_b \\ c_u = \dfrac{q_u}{2} \text{ of } \dfrac{s_u}{2} \end{array}}$$

The Beta Method

In beta method, the friction capacity is estimated as a fraction of the average effective vertical stress (as evaluated halfway down the pile)

$$Q_f = \beta\, p_{eff} PL$$
$$Q_b = N_q \left(p_{eff}\right)_b A_b$$

CAPACITY OF PILE GROUP

Some piles are installed in groups, spaced approximately 4 to 3.5 times the pile diameter apart. The files function as a group due to the use of the concrete load-transfer cap encasing all of the pile heads. The weight of the cap subtracts from the gross group capacity. The capacity due to the pile cap resting on the ground (as a spread footing) is disregarded.

For cohesionless (granular) soils, the capacity of the pile group is taken as the sum of the individual capacities, although the actual capacity will be greater. In-situ tests should be used to justify any increase.

For cohesive soils, the group capacity is taken as the smaller of (a) the sum of the individual capacities and (b) the capacity assuming block action. The block action capacity is calculated assuming that the piles from a large pier whose dimensions are group's perimeter. The block depth, L, is the distance from the surface to the depth of the pile points. The width of the pile group as measured from the outside (not center) of the outermost piles.

$$\text{Perimeter, } p = 2(b + w)$$
$$\text{Area, } A_p = (b + w)^2$$

The average undrained shear strength, c_u, along the depth of the piles is used to calculate the skin friction capacity. The average undrained shear strength at the pile tips, c_{ub} , is used to calculate the end-bearing capacity.
The group capacity can be more or less than the sum of the individual pile capacities. The file group efficiency, is

$$\boxed{\eta_G = \frac{\text{Group capacity, } Q_{ug}}{\sum \text{individual capacities, } Q_u}}$$

TENSILE CAPACITY OF PILES

Tension piles are intended to resist upward forces Basement and buried tanks below water level may require tension piles to prevent “floating away”. However, tall buildings subjected to overturning moments also need to resist pull-out. Unlike piles loaded in compression, the pull-out capacity of piles does not include the tip capacity. The pullout capacity includes the weight of the pile and the shaft resistance (skin friction).

SETTLEMENT OF PILE GROUP

Piles bearing on rock essentially do not settle. Piles in sand experience minimal settlement. Piles in clay may experience significant setting. The settlement of a pile group can be estimated by assuming that the support block (used to calculate the group capacity) extends to a depth of only 2/3 of the pile length. Settlement above (2/3)L is assumed to be negligible. Below the (2/3)L depth, the pressure distribution spreads out at a vertical: horizontal rate of 2:1. The Presence of the lower L/3 pile length is disregarded.

SLOPE STABILITY

The maximum slope for cuts in cohesionless (drained) sand is the angle of internal friction or angle of repose, Ø. In cohesive soils such as clay however, the maximum slope for cuts is more difficult to determine.

The soil or rock in a slope exists in a state of equilibrium between gravity forces tending to move the material down the slope and the internal shearing resistance of the material. A slope failure occurs when the force tending to cause rupture exceeds the resisting force. The overstressing of a slope or reduction in shear strength may cause displacements that may be very slow or very rapid and progressive.

Extremely slow movements in soils are called soil creep Rapid movements of intact or nearly intact soil or rock masses are called slides. Rock or soil that detaches from a nearly vertical slope and descends mainly through the air by falling, bouncing, or rolling is called a fall. Very soft cohesive soils can fail by lateral spreading or by mud flows.
The factors to be considered for stability or slope are the cohesion of the soil, c, shear strength, τ, soil stratification and its in-place shear strength parameters. Seepage through the slope and the choice of potential slip surface and up to the complexity of the problem

FACTORS OF SAFETY

The primary purpose of analyzing slope stability is to determine the factor of safety. In general, factor of safety is the ratio the average shear strength of soil, τ, to the average shear stress developed along the potential failure surface.

$$FS = \frac{\tau}{\tau_d}$$

$$\tau = c + \sigma \tan \emptyset$$

$$\tau_d = c_d + \sigma \tan \emptyset_d$$

Factor of safety with respect to strength:

$$FS_s = \frac{\tau}{\tau_d} = \frac{c + \sigma \tan \emptyset}{c_d + \sigma \tan \emptyset_d}$$

Factor of safety with respect to cohesion:

$$FS_c = \frac{c}{c_d}$$

Factor of safety with respect to friction:

$$FS_{\emptyset} = \frac{\tan \emptyset}{\tan \emptyset_d}$$

Relation of FS_s, FS_c, FS_ϕ:

$$\boxed{FS_s = FS_c = FS_\phi}$$

When $F_s = 1$, the slope is in a state of impending failure

STABILITY OF INFINITE SLOPE WITHOUT SEEPAGE

Infinite slope analysis is used when a layer of firm soil or rock lies parallel to a thin layer of softer material and the potential slip surfaces are very long compared to their depth. This occurs when a rock surface is parallel to the slope and there is a thin layer of soil overlying the rock. In this analysis, the driving forces of the uphill wedges and the resisting forces of the downhill wedges are ignored, and only the remaining central wedge is considered.

Consider 1 m strip perpendicular to the paper

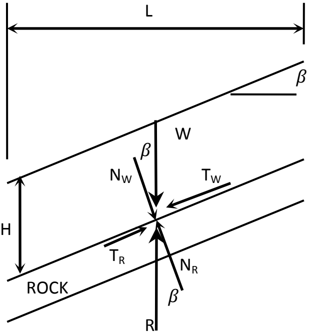


Figure: Analysis of infinite slope

Normal stress:

$$\sigma = \frac{N_w}{\text{Area of base}} = \frac{W\cos\beta}{(1)(L/\cos\beta)}$$

$$\sigma = \frac{\gamma V\cos\beta}{L/\cos\beta} = \frac{\gamma LH(1)\cos\beta}{L/\cos\beta}$$

$$\boxed{\sigma = \gamma H\cos^2\beta}$$

Tangential stress:

$$\tau = \frac{T_w}{\text{Area of base}} = \frac{W\sin\beta}{(1)(L/\cos\beta)}$$

$$\tau = \frac{\gamma V\sin\beta}{L/\cos\beta} = \frac{\gamma LH(1)\sin\beta}{L/\cos\beta}$$

$$\boxed{\begin{gathered}\tau = \gamma H\sin\beta\cos\beta \\[2ex] FS_s = \frac{c}{\gamma H\cos^2\beta\tan\beta} + \frac{\tan\phi}{\tan\beta}\end{gathered}}$$

If FS_s =1, H = critical depth, H_{cr}

$$1 = \frac{c}{\gamma H_{cr}\cos^2\beta\tan\beta} + \frac{\tan\phi}{\tan\beta}$$

$$\frac{\tan\beta - \tan\phi}{\tan\beta} = \frac{c}{\gamma H_{cr}\cos^2\beta\tan\beta}$$

$$H_{cr} = \frac{c}{\gamma}\frac{1}{\cos^2\beta(\tan\beta - \tan\varnothing)}$$

STABILITY OF INFINITE SLOPE WITH SEEPAGE

For soils with seepage and ground water level coincides with the ground surface:

$$\tau = c + \sigma' \tan\varnothing$$

$$\tau_d = c_d + \sigma' \tan\varnothing_d$$

Normal stress:

$$\sigma = \gamma_{sat} H\cos^2\beta$$

Effective stress:

$$\sigma' = \gamma' H\cos^2\beta = (\gamma_{sat} - \gamma_w) H\cos^2\beta$$

Tangential stress:

$$\tau = \gamma_{sat} H \sin\beta \cos\beta$$

$$FS_s = \frac{c}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma' \tan\varnothing}{\gamma_{sat} \tan\beta}$$

FINITE SLOPE WITH PLANE FAILURE (CULMANN'S METHOD)

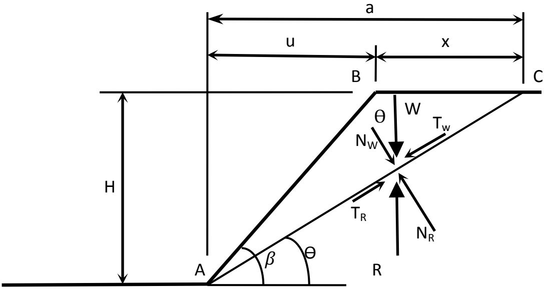


Figure: Finite slope with plane failure

Normal stress:

$$\sigma = \frac{1}{2}\,\gamma H\,\left[\frac{\sin(\beta - \theta)}{\sin\beta}\right]\cos\theta$$

Critical angle of slip plane:

$$\theta_{cr} = \frac{\beta + \phi_d}{2}\;;\; c_d = \frac{1}{4}\gamma H\left(\frac{1 - \cos(\beta - \phi_d)}{\sin\beta\cos\phi_d}\right)$$

This equation can also be written as

$$\frac{c_d}{\gamma H} = m = \frac{1 - \cos(\beta - \phi_d)}{4 \sin\beta \cos\phi_d}$$

Where m = stability number

The values of 1/m for various β and ϕ_d are shown in the following table.

Stability Numbers Based on Culmann's Analysis

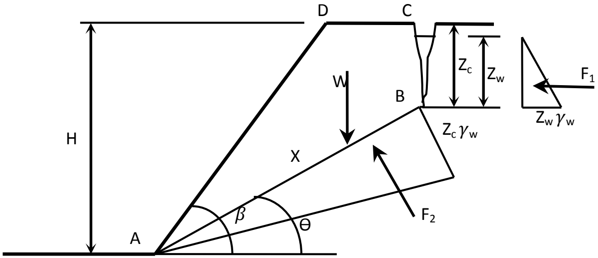
β (°)	ϕ_d	1/m	β (°)	ϕ_d	1/m	β (°)	ϕ_d	1/m	β (°)	ϕ_d	1/m
10	0	45.72	30	10	32.66	50	25	29.64	80	0	4.77
	5	181.84		15	56.70		30	44.00		5	5.29
15	0	30.38		20	123.71	60	0	6.93		10	5.90
	5	67.89		25	476.34		5	8.09		15	6.59
	10	267.93	40	0	10.99		10	9.55		20	7.40
20	0	22.69		5	14.16		15	11.42		25	8.37
	5	40.00		10	18.90		20	13.91		30	9.55
	10	88.68		15	26.51		25	17.36	90	0	4.00
	15	347.27		20	40.06		30	22.39		5	4.37
25	0	18.04		25	68.39	70	0	5.71		10	4.77
	5	27.92		30	146.57		5	6.49		15	5.21
	10	48.86	50	0	8.58		10	7.40		20	5.71
	15	107.48		5	10.42		15	8.51		25	6.28
	20	417.45		10	12.90		20	9.89		30	6.93
30	0	14.93		15	16.37		25	11.63			
	5	21.27		20	21.49		30	13.91			

When $c_d = c$ and $\phi_d = \phi$, then $H = H_{cr}$,

$$H_{cr} = \frac{4c}{\gamma}\left(\frac{\sin\beta \cos\phi}{1 - \cos(\beta - \phi)}\right)$$

SLOPES WITH WATER IN THE TENSILE CRACK:

When tensile cracks are developed at the top of the slope and filled with water, the stability of such slope can be determined in the following manner



Finite slope with water on tensile crack

Z_c = depth of crack
Z_w = depth of water in the crack
X = length AB = $(H - Z_c)/\sin\Theta$
Θ = angle of failure plane
W = weight of soil wedge ABCD
F_1 = force due to water in the crack
F_2 = force due to pore water pressure along AB

$F_1 = \frac{1}{2}\gamma_w z_1^{\,2}$

$F_2 = \dfrac{1}{2}\gamma_w z_c X \; = \dfrac{1}{2}\gamma_w z_c\,(H - z_C\,)/\sin\theta$

Components of W and F1 along AB:

$$F = W\sin\theta + F_1\cos\theta$$

Resisting force to F:

$$R = cX + (W\cos\theta - F_1\sin\theta - F_2)\tan\phi$$

Factor of safety with respect to strength:

$$\boxed{FS_s = \frac{R}{F} = \frac{cX + (W\cos\theta - F_1\sin\theta - F_2)\tan\phi}{W\sin\theta + F_1\cos\theta}}$$

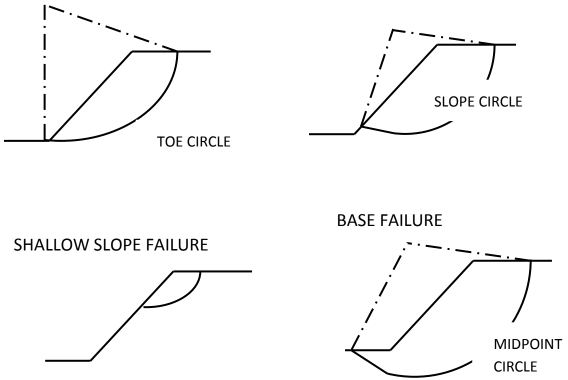
The magnitude of FS_s for the various trial wedges can be calculated by varying the value of θ. The minimum value of FS_s is the factor of safety of the slope.

ANALYSIS OF THE FINITE SLOPES WITH CIRCULAR FAILURE SURFACES – GENERAL FAILURE SURFACES:

Modes of failure:

Generally, finite slope failure occurs in one of the following diagrams:

SLOPE FAILURES



Two major classes of stability analysis procedure:

1. Mass Procedure – the soil that formed the slope is assumed to be homogeneous and the mass of soil above the surface of sliding is taken as a unit
2. Method of Slices – in this procedure, the non homogeneity of the soil and the pore water pressure can be taken into consideration and the soil above the surface of sliding is divided into a number of vertical parallel slices.

MASS PROCEDURE
(Homogeneous Clay Soil with ∅ = 0)

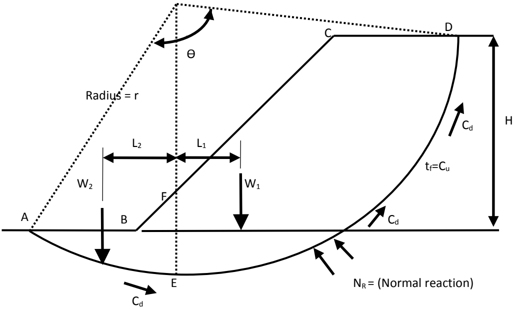


Figure: Stability analysis in homogeneous clay(Θ = 0)

W_1 = (Area of FCDEF) γ
W_2 = (Area of ABFEA) γ

Driving force about O to cause instability:
$M_D = W_1L_1 - W_2L_2$
Developed cohesion along the surface of sliding
$M_R = c_d(\text{arcAED})(1)r$
$M_R = c_d r^2 \theta$

For equilibrium:

$M_R = M_D$

$c_d r^2 \theta = W_1 L_1 - W_2 L_2$

$$c_d = \frac{W_1 L_1 - W_2 L_2}{r^2 \theta}$$

Factor of safety against sliding:

$$\boxed{FS_s = \frac{\tau}{c_d} = \frac{c_u}{c_d}}$$

For critical circles

$$\boxed{c_d = \gamma H m \text{ or } m = \frac{c_d}{\gamma H}}$$

For critical height; $FS_s = 1$, $H = H_{cr}$ and $c_d = c_u$

$$\boxed{H_{cr} = \frac{c_u}{\gamma m}}$$

Where m = stability number

c_u = undrained shear strength

c_d = developed cohesion

<u>TAYLOR SLOPE STABILITY CHART</u>

For saturated clay with slope of zero, the Taylor slope stability chart can be used to determine the factor of safety against slope failure. The Taylor chart makes the following assumptions:

(a) There is no water outside the slope

(b) There is no surcharge or tension cracks
(c) Shear strength is derived from cohesion only and is constant with depth
(d) Failure takes place as rotation on a circular arc

$$FS_S = N_o \frac{c}{\gamma' H}$$

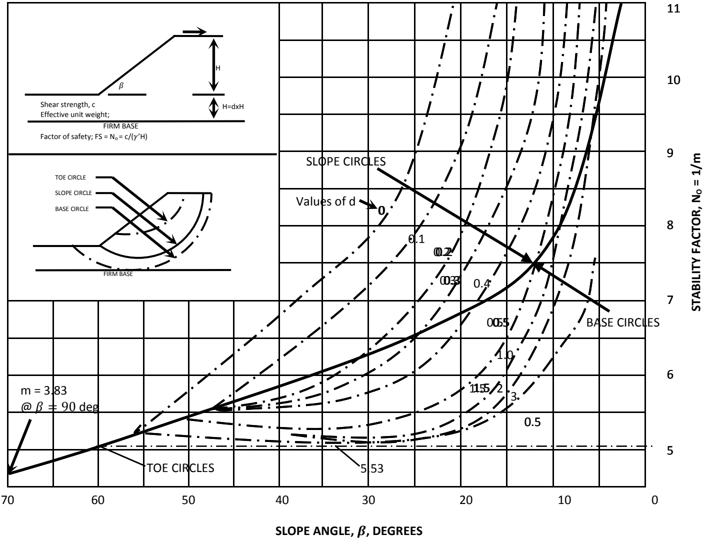
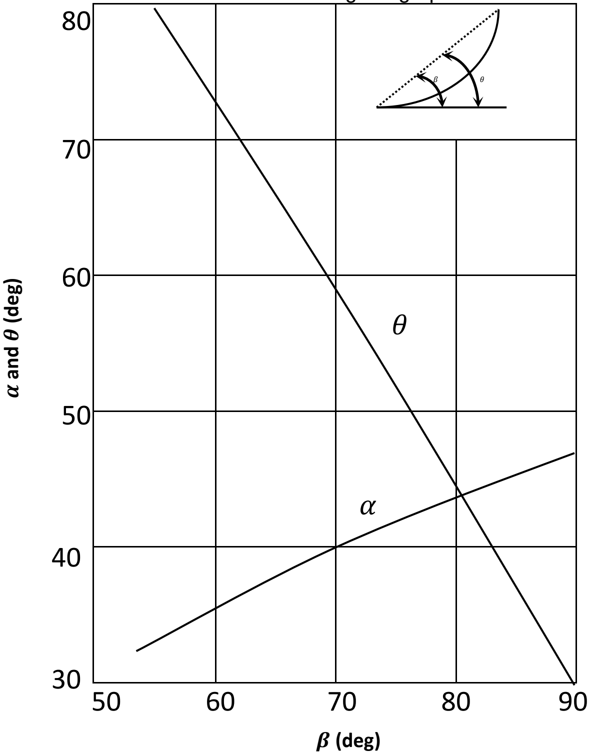


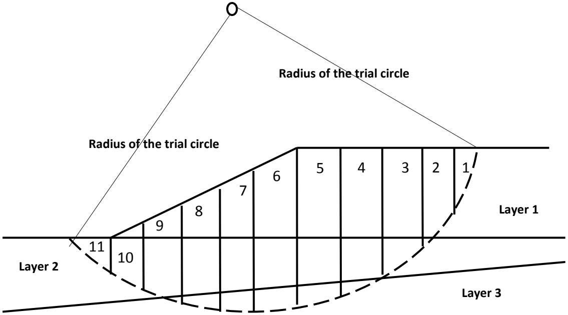
Figure: Taylor Slope Stability Chart

The Taylor chart shows that toe circle failures occur in slopes steeper than 53 degree. For slopes less than 53 degree, slope circle failure, toe circle failure, or base circle failure may occur. For $\beta > 53°$, all circles are toe circles. The location of the center of critical circle can be found using the graph below



METHOD OF SLICES

The method of slices was developed in the early 1920s in Sweden and was later refined by Bishop to consider interslice forces to some degree. This analysis method can accommodate complex slope geometries, variable soil layering and strengths, variable pore water pressure conditions, internal reinforcement, and the influence of external boundary loads, but it is only applicable to circular slip surfaces. It accomplishes this by dividing a slope into a series of vertical slices for analysis, with limiting equilibrium conditions evaluated for each slice, as shown



Typical slope stability analysis using the method of slices

Each slice can have different layering, different strength, and different pore water pressure than the other slices. If the condition of equilibrium is satisfied for each slice, then it is considered that the entire mass is in equilibrium. The force system on a single slice is shown

$$FS = \frac{\sum_{n=1}^{n=p} (cb_n + w_n \tan \emptyset) \frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin \alpha_n}$$

$$m_{\alpha(n)} = \cos \alpha_n + \frac{\tan\emptyset \sin \alpha_n}{FS}$$

Note that the FS is present on both sides of the equation. Hence, a trial-and error solution or a programmable calculator is necessary to find the value of FS.

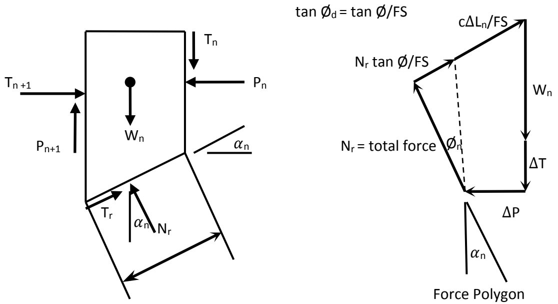


Figure: Forces acting on the slice on the nth slice in the Bishop simplified method of slices